Name Answer Key

Period __________ Date ______________

Calculus 4.1-4.5 Review

Solve each problem.

1. The length of a rectangle is increasing at a rate of 8 cm/s and the width is increasing at a rate of 3 cm/s. When the length is 20 cm and the width is 10 cm , how fast is the area changing?

$\frac{dl}{dt} = 8cm/s$ $\quad \frac{dw}{dt} = 3cm/s$ $\quad l = 20 \quad w = 10$

$A = l \cdot w$

$\frac{dA}{dt} = \frac{dw}{dt} l + \frac{dl}{dt} w$ $\qquad \frac{dA}{dt} = \underbrace{3(20)}_{60} + \underbrace{8(10)}_{80}$

$\boxed{\frac{dA}{dt} = 140 cm^2/s}$

2. The volume, V, of a cylinder with diameter 10 cm and height x is increasing at a rate of 100 cm^3/s, what is the rate of change of the height of the cylinder when the volume is 250π cm^3?

$V = \pi r^2 h$ $\qquad r = 5cm$ $\qquad \frac{dh}{dt}$ $\quad \frac{dV}{dt} = 100cm^3/s$

$V = \pi 25 h$

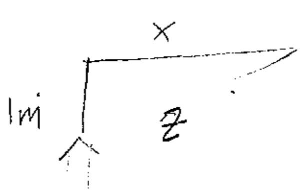
$\frac{dV}{dt} = 25\pi \frac{dh}{dt}$ $\qquad 100 = 25\pi \frac{dh}{dt}$ $\qquad \boxed{\frac{dh}{dt} = \frac{4}{\pi} cm/s}$

3. A plane flying horizontally at an altitude of 1 mi and a speed of 500 mi/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 mi away from the station.

$z = 2$

$x = \sqrt{3}$

$\frac{dx}{dt} = 500mi/h.$



$x^2 + 1^2 = z^2$

$2x \frac{dx}{dt} = 2z \frac{dz}{dt}$

$\cancel{2}\sqrt{3}(500) = \cancel{2}(2) \frac{dz}{dt}$

$\boxed{\frac{dz}{dt} = 250\sqrt{3} \ mi/hr}$

4. If the volume of a cube is increasing at a rate of 400 cm^3/s, what is the rate of change of a side length when the volume is 27 cm^3 ?

$V = x^3$ $\quad \frac{dV}{dt} = 400 \text{ cm}^3/s$ $\quad \frac{dx}{dt}$? $\quad V = 27 \quad x = 3$

$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$ $\quad 400 = 3(9)\frac{dx}{dt}$ $\quad \boxed{\frac{dx}{dt} = \frac{400}{27} \text{ cm/s}}$

5. Find the critical numbers of the function and then find the absolute maximum and minimum values of the function $f(x) = 3x^2 - 12x + 5$ over the interval [0, 3].

$f'(x) = 6x - 12$ $\quad f(0) = 5$ max

C#'s are 0, 3, 2 $\quad f(2) = -7$ min

$f(3) = -4$

6. . Given the function $f(x) = 5 + 3x^5 - 5x^3$, find:

a. intervals of increase or decrease.

b. intervals of concavity and the inflection points.

c. local maximum and minimum values.

d. Sketch the graph of f.

a) $f'(x) = 15x^4 - 15x^2$ $\quad (-\infty,-1)(-1,0)(0,1)(1,\infty)$ $\quad \uparrow \; \downarrow \; \downarrow \; \uparrow$

$f'(x) = 15x^2(x^2-1)$ $\quad$ local max $f(-1) = 7$

$f'(x) = 15x^2(x-1)(x+1)$ $\quad$ local min $f(1) = 3$

C#'s 0, 1, -1

b) $f''(x) = 60x^3 - 30x$

$f''(x) = 15x(4x^2 - 2)$

$x^2 = \frac{1}{2}$ $\quad x = \pm\sqrt{1/2}$

$0, \pm\sqrt{1/2}$

$(-\infty, -\sqrt{1/2})(-\sqrt{1/2}, 0)(0, \sqrt{1/2})(\sqrt{1/2}, \infty)$

CD $\quad$ CU $\quad$ CD $\quad$ CU

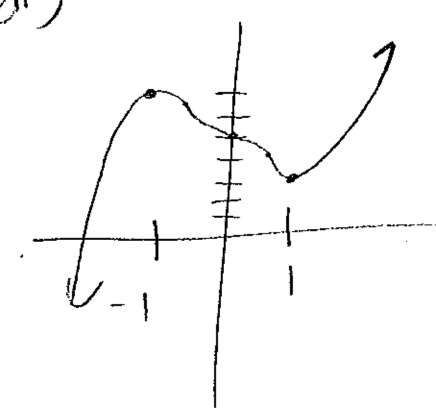
c)

IP $f(\sqrt{1/2}) = 3.76$

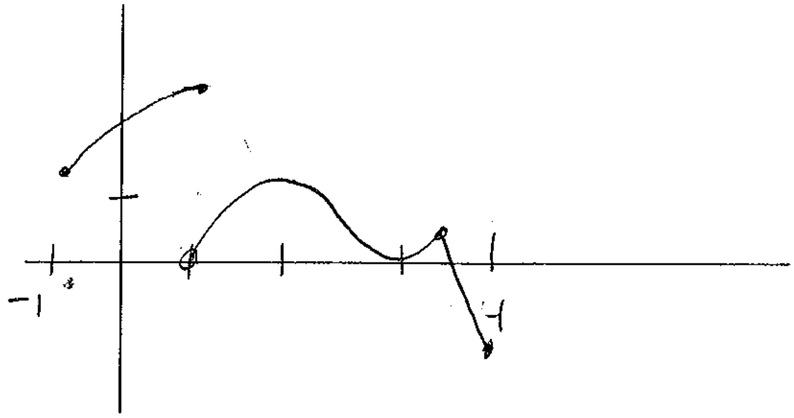
$f(0) = 5$

$f(-1/\sqrt{2}) = 6.237$

d)



7. Sketch the graph of the function that has an absolute maximum at 1, absolute minimum at 4, a local maximum at 2, and local minimum at 3 over the interval [-1, 4].



Use L'Hospital's Rule to evaluate each limit. State the indeterminate form that justifies its use.

8. $\lim_{t \to 0} \frac{5^t - 3^t}{t}$ $\quad \frac{1-1}{0} = \frac{0}{0}$

$\overset{H}{=} \frac{5^t \ln 5 - 3^t \ln 3}{1}$

$\ln 5 - \ln 3$

or $\boxed{\ln \frac{5}{3}}$

9. $\lim_{x \to 1} \frac{\ln x}{\sin \pi x} = \frac{0}{0}$

$\overset{H}{=} \frac{\frac{1}{x}}{\pi \cos \pi x} = \frac{1}{\pi x \cos \pi x}$

$-\frac{1}{\pi}$

10. $\lim_{x \to 0} \frac{\cos mx - \cos nx}{x^2}$ $\quad \frac{1-1}{0} = \frac{0}{0}$

$\overset{H}{=} \frac{-m \sin mx + n \sin nx}{2x} = \frac{0+0}{0} = \frac{0}{0}$

$\overset{H}{=} \frac{-m^2 \cos mx + n^2 \cos nx}{2} = \boxed{\frac{-m^2 + n^2}{2}}$

11. $\lim_{x \to \infty} \frac{e^{4x} - 1 - 4x}{x^2}$ $\quad \frac{\infty - 1 - \infty}{\infty} = \frac{\infty}{\infty}$

$\overset{H}{=} \frac{4e^{4x} - 4}{2x} = \frac{\infty}{\infty}$

$\overset{H}{=} \frac{16e^{4x}}{2} = 8 \cdot \infty = \infty$

12. Compare and Contrast local max and mins with absolute max and mins.

abs can be endpoints
one value

loc cannot be endpoints
several

all must be defined