Name Answer Key
Date _____ Period _____

Calculus Review 3.6-3.8

Find the derivative of each function.

1. $y = \log_6(xe^x)$

$\frac{g'(x)}{g(x)\ln 6}$ $f = x$ $g = e^x$ $f' = 1$ $g' = e^x$

$\frac{e^x - xe^x}{xe^x \ln 6} = \boxed{\frac{1+x}{x\ln 6}}$

2. $f(t) = t\ln(3+e^t)$

$f = t$ $g = \ln(3+e^t)$ $f' = 1$ $g' = \frac{e^t}{3+e^t}$

$\boxed{f'(t) = \ln(3+e^t) + \frac{te^t}{3+e^t}}$

3. $y = \ln\frac{(2t+1)^3}{(3t-1)^4}$

$y = 3\ln(2t+1) - 4\ln(3t-1)$

$y' = \frac{3(2)}{2t+1} - \frac{4(3)}{3t-1}$

$\boxed{y' = \frac{6}{2t+1} - \frac{12}{3t-1}}$

4. $y = \log_2(1-3x)$

$\frac{g'(x)}{g(x)\ln 2}$

$\boxed{y' = \frac{-3}{(1-3x)\ln 2}}$

5. $y = \ln\left(\frac{(2x^2+1)^4(3x^2)}{(4x^5-x)}\right)$

$y = 4\ln(2x^2+1) + \ln 3x^2 - \ln(4x^5-x)$

$y' = \frac{4(4x)}{2x^2+1} + \frac{6x}{3x^2} - \frac{20x^4-1}{4x^5-x}$

$\boxed{y' = \frac{16x}{2x^2+1} + \frac{2}{x} - \frac{20x^4-1}{4x^5-x}}$

6. $y = \cos^{-1}(3x^2+1)$

$y = \arccos u$ $u = 3x^2+1$ $u' = 6x$

$\frac{-1}{\sqrt{1-u^2}}$

$\frac{-1}{\sqrt{1-(3x^2+1)^2}}$

$\boxed{y' = \frac{-6x}{\sqrt{1-(3x^2+1)^2}}}$

7. $y = \sqrt{x}e^{5^x}(x^5+1)^{10}$

$\ln y = \frac{1}{2}\ln x + 5^x + 10\ln(x^5+1)$

$\frac{y'}{y} = \frac{1}{2x} + 5^x \ln 5 + \frac{10(5x^4)}{x^5+1}$

$y' = \left[\frac{1}{2x} + 5^x \ln 5 + \frac{50x^4}{x^5+1}\right]\left(\sqrt{x}\, e^{5^x}(x^5+1)^{10}\right)$

8. $y = \frac{(5x^2-2x)^5}{x^3}$

$\ln y = 5\ln(5x^2-2x) - 3\ln x$

$\frac{y'}{y} = \frac{5(10x-2)}{5x^2-2x} - \frac{3}{x}$ $\Rightarrow$ $\left[\frac{50x-10}{5x^2-2x} - \frac{3}{x}\right]\left(\frac{(5x^2-2x)^5}{x^3}\right)$

9. $y = \left(\sqrt{x+1}\right)^{\cos x}$

$\ln y = \cos x \ln(x+1)^{1/2}$

$\ln y = \frac{1}{2}\cos x \ln(x+1)$

$f = \frac{1}{2}\cos x \quad g = \ln(x+1)$

$f' = -\frac{1}{2}\sin x \quad g' = \frac{1}{x+1}$

$\frac{y'}{y} = -\frac{1}{2}\sin x(\ln(x+1)) + \frac{\cos x}{2x+2}$

$y' = \left(\frac{-\sin x(\ln(x+1))}{2} + \frac{\cos x}{2x+2}\right)\left(\sqrt{x+1}\right)^{\cos x}$

10. $y = \arctan(x^3+1.5x^2)$

$\arctan u \quad u = x^3+1.5x^2$

$u' = 3x^2+3x$

$\frac{1}{1+u^2}$

$\frac{1}{1+(x^3+1.5x^2)^2}$

$y' = \frac{3x^2+3x}{1+(x^3+1.5x^2)^2}$

Also, be prepared to answer questions about:

- orthogonal curves,
- evaluate positions of particles and distances traveled by the particles,
- be able to state when a particle is speeding up and when it is slowing down based on a position function, a velocity graph, and/or a position function graph,
- and be able to find the Marginal Cost of a function and explain its meaning.