2012-2013

Section 5.5

The Substitution Rule: If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I, then

$$\int f(g(x))g'(x)\ dx = \int f(u)\ du$$

Example 1: Find $\int 2x\sqrt{1+x^2}\,dx$.

so $u = 1+x^2$

$\frac{du}{dx} = 2x$ $\quad du = 2x\,dx$

$\int \sqrt{u}\ du$

$= \int u^{1/2}\ du = \frac{2u^{3/2}}{3} + C$

$\boxed{\frac{2(1+x^2)^{3/2}}{3} + C}$

Example 2: Find $\int x^3\cos(x^4+2)dx$.

$u = x^4+2$

$du = 4x^3\,dx$

$\frac{1}{4}du = x^3\,dx$

$\frac{1}{4}\int \cos u\ du$

$\frac{1}{4}\sin u + C$

$\boxed{\frac{\sin(x^4+2)}{4} + C}$

Example 3: Evaluate $\int \sqrt{1+2x}\ \ dx$.

$u = 1+2x$

$du = 2\ dx$

$\frac{1}{2}\,du = dx$

$$\frac{1}{2}\int u^{1/2}\,du$$

$$\frac{1}{2}\,\frac{2u^{3/2}}{3} + c = \boxed{\frac{(1+2x)^{3/2}}{3} + c}$$

Example 4: Evaluate $\int \frac{x}{\sqrt{1-4x^2}}\,dx$.

$u = 1-4x^2$

$du = -8x\ dx$

$-\frac{1}{8}\,du = x\ dx$

$$-\frac{1}{8}\int \frac{1}{\sqrt{u}}\,du$$

$$= -\frac{1}{8}\int u^{-1/2}\,du = -\frac{1}{8}\,2u^{1/2} + c$$

$$\boxed{\frac{-\sqrt{1-4x^2}}{4} + c}$$

Example 5: Calculate $\int e^{5x}dx$.

$u = 5x$

$du = 5\ dx$

$\frac{1}{5}\,du = dx$

$$\frac{1}{5}\int e^{u}\,du$$

$$\frac{1}{5}\,e^{u} + c$$

$$\boxed{\frac{e^{5x}}{5} + c}$$

5.5 Day two **Definite Integrals.**

The Substitution Rule for Definite Integrals: If g' is continuous on [a, b] and f is continuous on the range of $u = g(x)$, then

$$\int_a^b f(g(x))g'(x)\ dx = \int_{g(a)}^{g(b)} f(u)\ du$$

Example 1: Find $\int_0^4 \sqrt{2x+1}\ dx$ using evaluation theorem and substitution.

$u = 2x+1$

$du = 2\ dx$

$\frac{1}{2}\ du = dx$

$\frac{1}{2}\int u^{1/2}\ du = \frac{1}{2}\frac{2u^{3/2}}{3} + c$

$= \frac{u^{3/2}}{3} \Rightarrow \frac{(2x+1)^{3/2}}{3}\Big]_0^4 \Rightarrow \frac{27}{3} - \frac{1}{3} = \frac{26}{3} = 8\frac{2}{3}$

$2(4)+1 = 9$

$2(0)+1 = 1$

$\frac{1}{2}\int_1^9 u^{1/2}\ du$

$\frac{u^{3/2}}{3}\Big]_1^9 = \frac{9^{3/2}}{3} - \frac{1^{3/2}}{3} = 9 - \frac{1}{3} = 8\frac{2}{3}$

Example 2: Evaluate $\int_1^2 \frac{dx}{(3-5x)^2}$.

$u = 3-5x$

$du = -5\ dx$

$-\frac{1}{5}\ du = dx$

$a = -2$

$b = -7$

$-\frac{1}{5}\int_{-2}^{-7} u^{-2}\ du$

$+\frac{1}{5}\frac{1}{u}\Big]_{-2}^{-7} = \frac{1}{5}\left[\frac{-1}{7} + \frac{1}{2}\right]$

$\frac{1}{5}\left[\frac{-2}{14} + \frac{7}{14}\right] = \frac{1}{14}$

Example 3: Calculate $\int_1^e \frac{\ln x}{x}\,dx$.

$u = \ln x$

$du = \frac{1}{x}\,dx$

$a = 0 \quad b = 1$

$\int_0^1 u\,du$

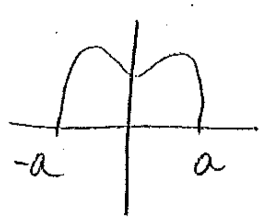
$\frac{u^2}{2}\Big]_0^1 = \frac{1}{2}$

Integrals of Symmetric Functions: Suppose f is continuous on $[-a, a]$.

a. If f is even $[f(-x)=f(x)]$, then $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$.

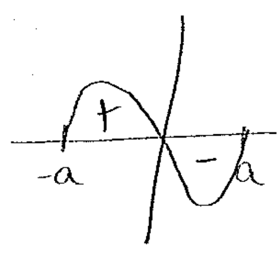
b. If f is ~~even~~ odd $[f(-x)=-f(x)]$, then $\int_{-a}^{a} f(x)\,dx = 0$.

a



$2\ (0, a)$

b



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Homework: day 1 page 381 (1-7, 9)
day 2 page 381 (8, 11-21 odd, 42, 43, 45,)