2012-2013

Section 5.4

The Fundamental Theorem of Calculus:
Part 1: If f is continuous on $[a, b]$, then the function g defined by

$$g(x) = \int_a^x f(t)\,dt \qquad a \le x \le b$$

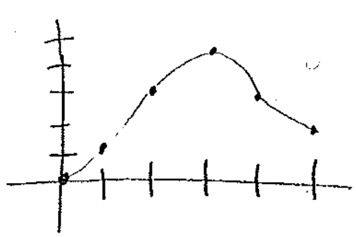
is an antiderivative of f, that is, $g'(x) = f(x)$ for all $a < x < b$.

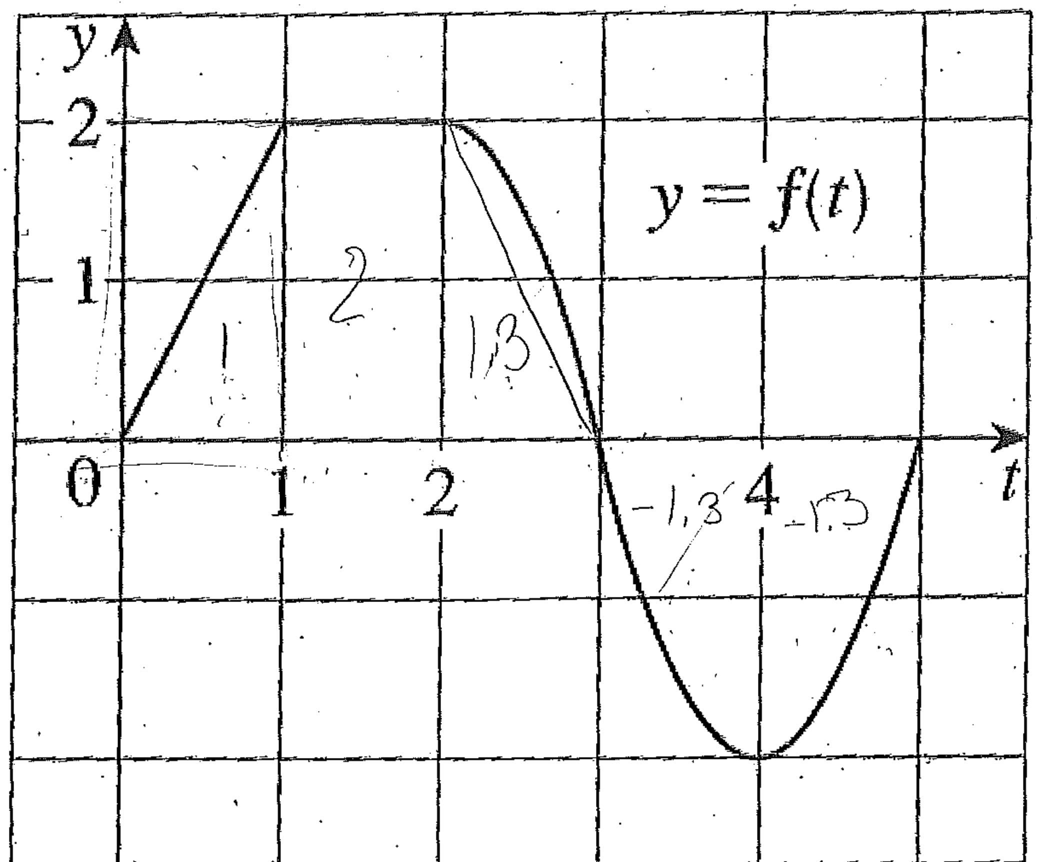
Part 2: $\int_a^b f(x)dx = F(b) - F(a)$, where F is any antiderivative of f, that is, $F' = f$.

Example 1: Let $g(x) = \int_a^x f(t)\,dt$, where f is the function whose graph is shown.
a. Evaluate $g(x)$ for $x = 0, 1, 2, 3, 4$, and 5.
b. Where does g have a maximum value? A minimum value?
c. Sketch a rough graph of g.

a) $g(0) = 0$ $g(1) = 1$ $g(2) = 3$ $g(3) = 4.3$ $g(4) = 3$
$g(5) = 1.7$

b) ↑↓ $g(3)$ max
$g(0)$ min

c) 



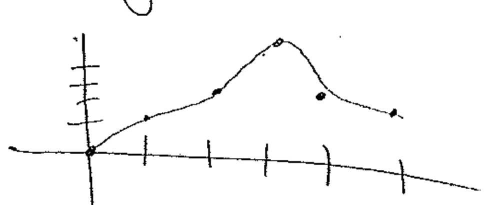
$g(0) = \int_0^0 g'(x)\,dx = 0 \quad g(1) = \int_0^1 = 1$

$g(2) = 3 \quad g(3) = 4.3 \quad g(4) = 3$

$g(5) = 1.7$

(b) max $g(3)$ min $g(0)$

(c)



Example 2: Find $g'(x)$ by using part 1 of FTC and then by part 2 of FTC with differentiation if

a. $g(x) = \int_{1}^{x} t^2 dt$

① $g'(x) = x^2$

② $g(x) = \frac{t^3}{3}\Big]_1^x = \frac{x^3}{3} - \frac{1}{3} = g(x)$

$\Rightarrow g'(x) = \frac{3x^2}{3} = \boxed{x^2}$

b. $g(x) = \int_{0}^{x} (1+t^2)\ dt$

① $g'(x) = 1 + x^2$

② $g(x) = t + \frac{t^3}{3}\Big]_0^x = x + \frac{x^3}{3} = g(x)$

$\Rightarrow g'(x) = 1 + \frac{3x^2}{3} = \boxed{1 + x^2}$

Example 3: Using part 1 of FTC find $g'(x)$ if

$g(x) = \int_{0}^{x} \sqrt{1+t^2}\, dt$.

$g'(x) = \sqrt{1+x^2}$

$\int_x^0 \sqrt{1+t^2}\ dt$

$= -\int_0^x \sqrt{1+t^2}\ dt = -\sqrt{1+t^2}$

Example 4: Find $\frac{d}{dx}\int_{1}^{x^4} \sec t \, dt$. use substitution

$u = x^4 \Rightarrow \frac{du}{dx} = 4x^3 \Rightarrow \frac{d}{dx} = \frac{d}{du}\left(\frac{du}{dx}\right)$

then $\frac{d}{dx}\int_{1}^{x^4} \sec t \, dt = \left[\frac{d}{du}\int_{1}^{u} \sec t \, dt\right]\frac{du}{dx}$

$= (\sec u)\frac{du}{dx}$

$= (\sec u)\, 4x^3 \Rightarrow \boxed{4x^3 \sec(x^4)}$

Example 4: Use part one of the FTC to find the derivative of $F(x) = \int_{x}^{3} \sin\theta \, d\theta$.

$F(x) = -\int_{3}^{x} \sin\theta \, d\theta$

$\boxed{F'(x) = -\sin x}$

Homework day 1 page 373 (1-13 odd), day 2 page 373 (2-14 even, 19, 23, 25)