Section 5.3

Evaluation Theorem: If f is continuous on the interval $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Where F is any antiderivative of f, that is, $F' = f$.

Indefinite Integral: $\int f(x)\,dx = F(x)$ means $F'(x) = f(x)$. (In other words the antiderivative with the + C on the end...)

Example 1: Evaluate $\int_1^4 e^x\,dx$.

$$e^x\Big]_1^4 = e^4 - e^1 = e^4 - e$$

Example 2: Find the general indefinite integral $\int (10x^4 - 2sec^2x)\;dx$.

$$2x^5 - 2\tan x + C$$

Example 3: Evaluate $\int_0^3 (x^3 - 6x)\;dx$.

$$\frac{x^4}{4} - 3x^2\Big]_0^3 = \left(\frac{3^4}{4} - 3(3)^2\right) - (0)$$

$$= \boxed{-6.75}$$

Example 4: Find $\int_0^2 \left(2x^3 - 6x + \frac{3}{x^2+1}\right)\, dx$.

$$\frac{x^4}{2} - 3x^2 + 3\arctan x \Big]_0^2$$

$$= \frac{2^4}{2} - 3(2)^2 + 3\arctan(2) - \underbrace{[0 - 0 + 3\arctan 0]}_{0}$$

$$= 8 - 12 + 3\arctan 2 = \boxed{-4 + 3\arctan 2}$$

Example 5: Evaluate $\int_1^9 \left(\frac{2t^2+t^2\sqrt{t}-1}{t^2}\right) dt$ $\quad x^{1/2} \quad \frac{2x}{3}^{3/2}$

$$= \int_1^9 (2 + \sqrt{t} - t^{-2})\, dt = 2t + \frac{2}{3}t^{3/2} + t^{-1}\Big]_1^9$$

$$2(9) + \frac{2}{3}(9)^{3/2} + 9^{-1} - \left[2 + \frac{2}{3} + 1\right]$$

$$18 + 18 + \frac{1}{9} - 3\tfrac{2}{3} = \boxed{32\tfrac{4}{9}}$$

Net Change Theorem: The integral of a rate of change is the net change: $\int_a^b F'(x)\, dx = F(b) - F(a)$

Example 6: A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in m/s).

a. Find the displacement of the particle during the time period [1, 4].

$$\int_1^4 t^2 - t - 6\, dt = \frac{t^3}{3} - \frac{t^2}{2} - 6t\Big]_1^4 = \boxed{-9/2}$$

b. Find the distance traveled during this time period. moving + vs moving −

$t^2 - t - 6 = 0$ $\quad$ $t = 3$ is in [1, 4) so [1, 3] + [3, 4].

$(t-3)(t+2) = 0$

$t = 3, -2$ $\quad$ so $-\int_1^3 t^2 - t - 6\, dt + \int_3^4 t^2 - t - 6\, dt$

$\approx \boxed{10.17\text{ m}}$

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$s(4) - s(1) = \int_1^4 (t^2 - t - 6)\,dx$

displacement $= \frac{t^3}{3} - \frac{t^2}{2} - 6t \Big]_1^4$

$= \frac{64}{3} - \frac{16}{2} - 24 - \frac{1}{3} + \frac{1}{2} + 6 = -9/2$

b) $(t-3)(t+2) < 0$

$-$ when t $[1,3]$ $\quad + > 0$ on $[3,4]$

$(1,3)$ $-$

$(3,4)$ $+$

$\int_1^4 |f(x)| = -\int_1^3 (t^2 - t - 6)\,dt + \int_3^4 (t^2 - t - 6)\,dt$

$= -\left(\frac{t^3}{3} - \frac{t^2}{2} - 6t\right)\Big]_1^3 + \left(\frac{t^3}{3} - \frac{t^2}{2} - 6t\right)\Big]_3^4$

$= +\left(\cancel{9} + \cancel{\frac{9}{2}} + \cancel{18} + \frac{1}{3} - \frac{1}{2} + 6\right) + \left(\cancel{\frac{64}{3}} - 8 - 24 - \cancel{9} + \cancel{\frac{9}{2}} \cancel{+18}\right)$

$= -18 + 36 + 9 + \frac{65}{3} - \frac{1}{2} - 38$

$= -9 - 2 - \frac{1}{2} + \frac{65}{3} = -11.5 + 21\frac{2}{3} = 10.17m$