Name Answer Key
Date ______ Period J

Calculus Review 5.3 to 5.5

Evaluate the definite integral.

1. $\int_1^{27} \sqrt[3]{x^2}\ dx$

$\int_1^{27} x^{2/3}\,dx$

$\frac{3x^{5/3}}{5}\Big]_1^{27}$

$\frac{3(27)^{5/3}}{5} - \frac{3}{5}$

$\boxed{=145.2}$

2. $\int_0^5 (x-1)(5x+3)\ dx$

$\int_0^5 (5x^2-2x-3)\,dx$

$= \frac{5x^3}{3} - x^2 - 3x\Big]_0^5$

$= \left(\frac{5(5)^3}{3} - 5^2 - 3(5)\right) - (0)$

$\boxed{= 168\tfrac{1}{3}}$

3. $\int_1^e \frac{1}{x}\ dx$

$\ln x\Big]_1^e$

$\ln e - \ln 1$

$1 - 0$

$= 1$

4. $\int_{-\pi/6}^{\pi/6} \frac{3}{\sqrt{1-x^2}}\ dx$

$= 3\arcsin x\Big]_{-\pi/6}^{\pi/6}$

$= 3\arcsin \pi/6 - 3\arcsin(-\pi/6)$

$\boxed{\approx 3.3}$

5. $\int_1^3 \frac{x^2}{x^3+1}\ dx$

$u = x^3+1$
$du = 3x^2\,dx$
$\frac{1}{3}du = x^2\,dx$
$a = 2$
$b = 28$

$\frac{1}{3}\int_2^{28} u^{-1}\,du$

$\frac{1}{3}\ln u\Big]_2^{28}$

$\frac{1}{3}\ln 28 - \frac{1}{3}\ln 2$

$\boxed{\frac{1}{3}\ln 14 \approx .88}$

6. $\int_0^1 2y\sin(y^2)\ dy$

$u = y^2$
$du = 2y\,dy$

$\int_0^1 \sin u\,du$ $\quad a = 0$, $b = 1$

$-\cos u\Big]_0^1$

$-\cos 1 + \cos 0 = \boxed{.46}$

7. $\int_0^7 \sqrt{4+3x}\,dx$ $u = 4+3x$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$a = 4$

$b = 25$

$\frac{1}{3}\int_4^{25} u^{1/2}\,du$

$\frac{1}{3}\frac{2u^{3/2}}{3}\Big]_4^{25} = \frac{2}{9}u^{3/2}\Big]_4^{25}$

$= \frac{2}{9}(25)^{3/2} - \frac{2}{9}(4)^{3/2}$

26

8. $\int_0^{\sqrt{\pi}} x\cos(x^2)\,dx$

$u = x^2$ $du = 2x\,dx$ $a = 0$

$\frac{1}{2}du = x\,dx$ $b = \pi$

$\frac{1}{2}\int_0^{\pi} \cos u\,du$

$\frac{1}{2}\sin u\Big]_0^{\pi} = \frac{1}{2}\sin\pi - \frac{1}{2}\sin 0$

$= \boxed{0}$

Evaluate the indefinite integral.

9. $\int (2-x)^6 dx$ $u = 2-x$

$du = -x\,dx$

$-du = x\,dx$

$-\int u^6\,du$

$-\frac{u^7}{7} + C$

$\boxed{-\frac{(2-x)^7}{7} +}$

10. $\int \frac{x}{(x^2+1)^2}dx$ $u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\frac{1}{2}\int u^{-2}\,du$

$-\frac{1}{2}\frac{1}{u} + C$

$= \boxed{\frac{-1}{2(x^2+1)} + C}$

11. $\int \frac{1}{(5t+4)^{2.7}}dt$ $u = 5t+4$

$du = 5\,dt$

$\frac{1}{5}du = dt$

$\frac{1}{5}\int u^{-2.7}\,du$

$\frac{1}{5}\frac{u^{-1.7}}{-1.7} + C$

$= \boxed{\frac{-2}{17(5t+4)^{1.7}} + C}$

12. $\int y^3\sqrt{2y^4-1}\quad dy$ $u = 2y^4-1$

$du = 8y^3\,dy$

$\frac{1}{8}du = y^3\,dy$

$\frac{1}{8}\int u^{1/2}\,du$

$= \frac{1}{8}\frac{2u^{3/2}}{3} + C$

$= \boxed{\frac{(2y^4-1)^{3/2}}{12} + C}$

13. $\int \frac{x^5 - 2x^2 + \sqrt[3]{x}}{x}\, dx$

$= \int \left(x^4 - 2x + x^{-2/3}\right) dx$

$= \boxed{\frac{x^5}{5} - x^2 + 3x^{1/3} + C}$

14. $\int x^2\left(3x^5 + 2x^4 + \frac{1}{x^2}\right)\, dx$

$= \int (3x^7 + 2x^6 + 1)\, dx$

$= \boxed{\frac{3x^8}{8} + \frac{2x^7}{7} + x + C}$

15. $\int 3x^2\sqrt{5 + x^3}\, dx$

$u = 5 + x^3$

$du = 3x^2 dx$

$\int u^{1/2}\, du$

$\frac{2u^{3/2}}{3} + C$

$\boxed{\frac{2(5+x^3)^{3/2}}{3} + C}$

16. $\int x^4 \sin(x^5)\, dx$

$u = x^5$

$du = 5x^4$

$\frac{1}{5} du = x^4$

$\frac{1}{5}\int \sin u\, du$

$-\frac{1}{5} \cos u + C$

$\boxed{-\frac{\cos x^5}{5} + C}$

Find the derivative of the function.

17. $\int_{x^3}^{3} \cos\theta \; d\theta$

$= -\int_{3}^{x^3} \cos\theta \, d\theta$

$\boxed{= -3x^2 \cos x^3}$

18. $\int_{3}^{\sqrt{x}} \frac{\sin t}{t} \; dt$

$= \frac{\sin\sqrt{x}}{2\sqrt{x}\sqrt{x}} = \boxed{\frac{\sin\sqrt{x}}{2x}}$

19. Find $\frac{d}{dx}\int_{1}^{x^2} \sin t \; dt$.

$\boxed{= 2x \sin(x^2)}$