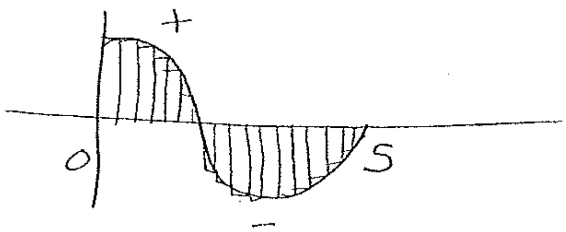
Section 5.2

Definition 3: $\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$

Riemann Sum: $\sum_{i=1}^{n} f(x_i)\Delta x$

Net Area: Riemann sums can be interpreted as the difference in Area when a graph is both above and below the x-axis.



Example 1: Express $\lim_{n\to\infty} \sum_{i=1}^{n} (x_i^3 + \sin x_i)\Delta x$ as an integral over the interval [0, π].

$\int_a^b f(x)\,dx \qquad \int_0^\pi (x^3+\sin x)\,dx$

Example 2: Evaluate the Riemann sum for $f(x) = x^3 - 6x$ taking the sample points to be right endpoints and a = 0, b = 3, and n = 6.

$\int_0^3 (x^3-6x)\,dx$

First $\Delta x = \frac{b-a}{n} = \frac{3-0}{6} = \frac{1}{2}$

$\cancel{0}, \frac{1}{2}, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}, 3$

$R_6 = \sum_{i=1}^{6} f(x_i)\Delta x = \frac{1}{2}(-2.875-5-5.625-4+.625+9)$

$R_6 = -3.9375$

Example 3: Evaluate $\int_0^3 (x^3 - 6x)dx$. $\Delta x = \frac{3-0}{n} = \frac{3}{n}$ $\Rightarrow x_i = \frac{3i}{n}$

$$\int_0^3 (x^3 - 6x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(\frac{3i}{n}\right)^3 - 6\left(\frac{3i}{n}\right)\right) \cdot \frac{3}{n}$$

$$\Rightarrow \frac{3}{n}\sum_{i=1}^{n} \frac{27i^3}{n^3} - \frac{3}{n}\sum_{i=1}^{n}\frac{18i}{n}$$

$$= \frac{81}{n^4}\sum_{i=1}^{n} i^3 - \frac{54}{n^2}\sum_{i=1}^{n} i$$

$$= \frac{81}{n^4}\left[\frac{n(n+1)}{2}\right]^2 - \frac{54}{n^2}\left(\frac{n(n+1)}{2}\right)$$

$$= \frac{81}{n^4}\left(\frac{n^2(n+1)^2}{4}\right) - \frac{27}{n}(n+1)$$

$$= \frac{81}{4n^2}\left((n+1)^2\right) - \frac{27}{n}(n+1)$$

$$= \frac{81}{4}\left(\frac{n+1}{n}\right)\left(\frac{n+1}{n}\right) - 27\left(\frac{n+1}{n}\right)$$

$$\Rightarrow \lim_{n\to\infty}\left(\frac{81}{4}\left(\frac{n}{n}+\frac{1}{n}\right)\left(\frac{n}{n}+\frac{1}{n}\right) - 27\left(\frac{n}{n}+\frac{1}{n}\right)\right)$$

$$= \frac{81}{4}(1)(1) - 27(1) = \boxed{-6\tfrac{3}{4}}$$

Example 4: Set up an expression for $\int_1^3 e^x dx$ as a limit of sums.

$\Delta x = \frac{3-1}{n} = \frac{2}{n}$ $\quad x_i = \frac{2i}{n} + 1$ because $1, \frac{2}{n}+1, \frac{4}{n}+1, \frac{6}{n}+1, \ldots$

$$\Rightarrow \lim_{n\to\infty}\sum_{i=1}^{n}\left(e^{\frac{2i}{n}+1}\right)\cdot\frac{2}{n}$$

$$= \lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n}\left(e^{\frac{2i}{n}+1}\right)$$

Example 5: Use the midpoint rule with $n = 5$ to approximate $\int_1^2 \frac{1}{x}\, dx$. $\Delta x = \frac{2-1}{5} = \frac{1}{5}$ or $.2$ x values are 1, 1.2, 1.3, 1.4, . . .

$\frac{1+1.2}{2} = 1.1$ so 1.1, 1.3, 1.5, 1.7, 1.9

$M_5 = .2(f(1.1) + f(1.3) + f(1.5) + f(1.7) + f(1.9))$

$M_5 = .2(.90909 + .76923 + .66667 + .58824 + .52632)$

$= \boxed{0.6919}$

Properties of Integrals:

1. $\int_a^b c\, dx = c(b-a)$, where c is any constant.

2. $\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)\, dx + \int_a^b g(x)\, dx$

3. $\int_a^b cf(x)\, dx = c\int_a^b f(x)\, dx$, where c is any constant

4. $\int_a^b [f(x) - g(x)]dx = \int_a^b f(x)\, dx - \int_a^b g(x)\, dx$

5. $\int_b^a f(x)\, dx = -\int_a^b f(x)\, dx$

6. $\int_a^a f(x)\, dx = 0$

7. $\int_a^c f(x)\, dx + \int_c^b f(x)\, dx = \int_a^b f(x)\, dx$

Homework: day 1 page 353 (1, 3, 5, 7, 9, 17, 19)

Day 2 page 353 (2, 6, 10, 11, 18, 20, 21, 23, 25, 27, 31)