Sec 5.2 day 1 page 353 (1, 3, 5, 7, 9, 17, 19)

① $f(x) = 3 - \frac{1}{2}x$ D: [2, 14] $\Delta x = \frac{14-2}{6} = 2$

$L_6 = \sum_{i=1}^{6} f(x_{i-1})\Delta x$

$= 2(f(2) + f(4) + f(6) + f(8) + f(10) + f(12))$
$= 2(2 + 1 + 0 + -1 + -2 + -3) = -6$

net area

③ $f(x) = e^x - 2$ D: [0, 2] $\Delta x = \frac{2-0}{4} = \frac{1}{2}$

mdpts $x_i^* = \bar{x}_i = \frac{1}{2}(x_{i-1} + x_i)$

$M_4 = \sum_{i=1}^{4} f(\bar{x}_i)\Delta x$ $0, \frac{1}{2}, 1, \frac{3}{2}, 2$
$\frac{1}{4}$ $\frac{3}{4}$ $\frac{5}{4}$ $\frac{7}{4}$

$M_4 = \frac{1}{2}(f(\frac{1}{4}) + f(\frac{3}{4}) + f(\frac{5}{4}) + f(\frac{7}{4})) \approx \boxed{2.322986}$

net area

⑤ $\int_0^8 f(x)\,dx$ $\Delta x = \frac{8-0}{4} = 2$

a) $R_4 = 2(f(2) + f(4) + f(6) + f(8)) = 2(1 + 2 + -2 + 1) = 4$
b) $L_4 = 2(f(0) + f(2) + f(4) + f(6)) = 2(2 + 1 + 2 + -2) = 6$
c) $M_4 = 2(f(1) + f(3) + f(5) + f(7)) = 2(3 + 2 + 1 + -1) = 10$

7. $L_5 = 4(-12 + -6 + -2 + 1 + 3) = -64$ lower

$R_5 = 4(-6 + -2 + 1 + 3 + 8) = 16$ upper

9. $\int_2^{10} \sqrt{x^3+1}\, dx$, $n = 4$

$\Delta x = \frac{10-2}{4} = 2$ 2, 4, 6, 8, 10

3 5 7 9

$M_4 = 2(f(3) + f(5) + f(7) + f(9))$

$M_4 = 2(\sqrt{28} + \sqrt{126} + \sqrt{344} + \sqrt{730}) \approx 124.1644$

17. $\int_2^6 (x \ln(1+x^2))\, dx$

19. $\int_1^8 \sqrt{2x + x^2}\, dx$

Sec 5.2 day 2 page 353 (2, 6, 10, 11, 18, 20, 21, 23, 25, 27, 31)

(2) $f(x) = x^2 - 2x$ $D: [0,3]$ $n=6$ $\Delta x = \frac{3-0}{6} = \frac{1}{2}$

$R_6 = \frac{1}{2}(f(\frac{1}{2}) + f(1) + f(\frac{3}{2}) + f(2) + f(\frac{5}{2}) + f(3)) =$
$= \frac{1}{2}(-\frac{3}{4} - 1 - \frac{3}{4} + 0 + \frac{5}{4} + 3) = \frac{7}{8}$

net area

(6) $\int_{-3}^{3} g(x)\,dx$ $\Delta x = \frac{3-(-3)}{6} = 1$

a) $R_6 = 1(f(-2) + f(-1) + f(0) + f(1) + f(2) + f(3)) =$
$= 1(1 - .5 - 1.5 - 1.5 - .5 + 2.5) = -.5$

b) $L_6 = 1(f(-3) + f(-2) + f(-1) + f(0) + f(1) + f(2)) =$
$= 1(2 + 1 - .5 - 1.5 - 1.5 - .5) = -1$

c) $M_6 = 1(f(-2\frac{1}{2}) + f(-1\frac{1}{2}) + f(-\frac{1}{2}) + f(\frac{1}{2}) + f(\frac{3}{2}) + f(\frac{5}{2})) =$
$= 1(1.5 + 0 - 1 - 1.75 - 1 + .5) = -1.75$

(10) $\int_0^{\pi/2} \cos^4 x\,dx$, $n=4$ $\Delta x = \frac{\pi/2 - 0}{4} = \pi/8$

$0,\ \pi/8,\ \frac{2\pi}{8},\ 3\pi/8,\ 4\pi/8$
$\pi/16 \quad 3\pi/16 \quad 5\pi/16 \quad 7\pi/16$ $M_4 = \pi/8(\cos^4(\pi/16) + \cos^4(\frac{3\pi}{16}) + \cos^4(\frac{5\pi}{16}) + \cos^4(\frac{7\pi}{16}))$

≈ 0.5890

(11) $\int_0^1 \sin(x^2)\,dx$, $n=5$ $\quad \Delta x = \frac{1-0}{5} = \frac{1}{5}$

$0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1$

$1/10, 3/10, 5/10, 7/10, 9/10$

$$M_5 = \frac{1}{5}\left(f(1/10) + f(3/10) + f(5/10) + f(7/10) + f(9/10)\right)$$
$$= \frac{1}{5}\left(\sin(\tfrac{1}{10})^2 + \sin(3/10)^2 + \sin(5/10)^2 + \sin(7/10)^2 + \sin(9/10)^2\right)$$
$$= 0.3084$$

(18) $\int_{\pi}^{2\pi} \frac{\cos x}{x}\,dx$

(20) $\int_0^2 (4-3x^2+6x^5)\,dx$

(21) $\int_{-1}^{5} (1+3x)\,dx$ $\quad \Delta x = \frac{5--1}{n} = \frac{6}{n}$

$x_i = \frac{6i}{n} - 1$

$$\Rightarrow \lim_{n\to\infty} \sum_{i=1}^{n} \left(1+3\left(\frac{6i}{n}-1\right)\right)\frac{6}{n} = \lim_{n\to\infty} \frac{6}{n}\sum_{i=1}^{n}\left(1+\frac{18i}{n}-3\right)$$

$$= \lim_{n\to\infty} \frac{6}{n}\sum_{i=1}^{n}\left(\frac{18i}{n}-2\right) = \lim_{n\to\infty}\left[\frac{108}{n^2}\sum_{i=1}^{n} i - \frac{12}{n}\sum_{i=1}^{n} 1\right]$$

$$= \lim_{n\to\infty}\left[\frac{108}{n^2}\left(\frac{n(n+1)}{2}\right) - \frac{12}{n}(n)\right] = \lim_{n\to\infty}\left[54\frac{(n+1)}{n} - 12\right]$$

$$= 54-12 = 42$$

(23) $\int_0^2 (2-x^2)\,dx \qquad \Delta x = \frac{2-0}{n} = \frac{2}{n}$

$x_i = \frac{2i}{n}$

$= \lim_{n\to\infty}\left[\sum_{i=1}^{n}\left(\left(2-\left(\frac{2i}{n}\right)^2\right)\frac{2}{n}\right)\right]$

$= \lim_{n\to\infty}\frac{2}{n}\left[\sum_{i=1}^{n} 2 - \frac{4i^2}{n^2}\right] = \lim_{n\to\infty}\left[\frac{4}{n}(n) - \frac{8}{n^3}\sum_{i=1}^{n} i^2\right]$

$= \lim_{n\to\infty}\left[4 - \frac{8}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)\right] = \lim_{n\to\infty}\left[4 - \frac{4}{3}\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right)\right]$

$= 4 - \frac{4}{3}(2) = 4 - \frac{8}{3} = 1\frac{1}{3}$

(25) $\int_1^2 x^3\,dx \qquad \Delta x = \frac{2-1}{n} = \frac{1}{n}$

$x_i = \frac{i}{n} + 1 \Rightarrow \lim_{n\to\infty}\left[\sum_{i=1}^{n}\left(\frac{i}{n}+1\right)^3\frac{1}{n}\right]$

$= \lim_{n\to\infty}\left[\boxed{\frac{1}{n}}\left(\sum_{i=1}^{n}\left(\frac{i^3}{n^3} + \frac{3i^2}{n^2} + \frac{3i}{n} + 1\right)\right)\right]$

$= \lim_{n\to\infty}\left[\frac{1}{n^4}\left[\frac{n(n+1)}{2}\right]^2 + \frac{3}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] + \frac{3}{n^2}\left[\frac{n(n+1)}{2}\right] + \frac{n}{n}\right]$

$= \lim_{n\to\infty}\frac{1}{4n^4}\left[n^2(n+1)^2\right] + \frac{1}{2}\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right) + \frac{3}{2}\left(\frac{n+1}{n}\right) + 1$

$= \frac{1}{4} + \frac{1}{2}(2) + \frac{3}{2} + 1 = 3\frac{3}{4}$

(27) $\int_2^6 \frac{x}{1+x^5}\,dx$ $\qquad \Delta x = \frac{6-2}{n} = \frac{4}{n}$

$x_i = \frac{4i}{n} + 2 \Rightarrow \boxed{\lim_{n\to\infty}\left(\frac{4}{n}\sum_{i=1}^{n} \frac{\frac{4i}{n}+2}{1+\left(\frac{4i}{n}+2\right)^5}\right)}$

(31) a) $\int_0^2 f(x)\,dx = 4$

b) $\int_0^5 f(x)\,dx = 10$

c) $\int_5^7 f(x)\,dx = -3$

d) $\int_0^9 f(x)\,dx = 10 + -3 + \int_7^9 f(x)\,dx =$

$10 - 3 - 5 = 2$