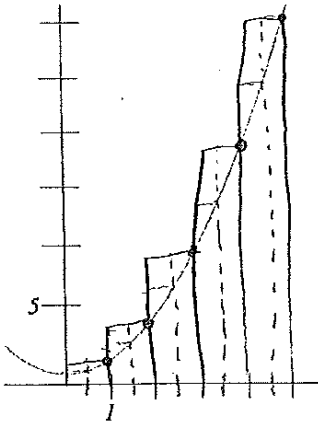


## Tec 5.1

Example 1: Estimate the area under the graph of  $f(x) = 1 + x^2$  from  $x = 0$  to  $x = 5$  using 5 rectangles and right endpoints. Now use 10 rectangles and right endpoints.



$$\begin{aligned} f(1) &= 2 \\ f(2) &= 5 \\ f(3) &= 10 \\ f(4) &= 17 \\ f(5) &= 26 \end{aligned}$$

$$\Delta x = 1$$

$$\sum_{i=1}^5 f(x_i) \Delta x$$

$$1(2 + 5 + 10 + 17 + 26) = \boxed{60}$$

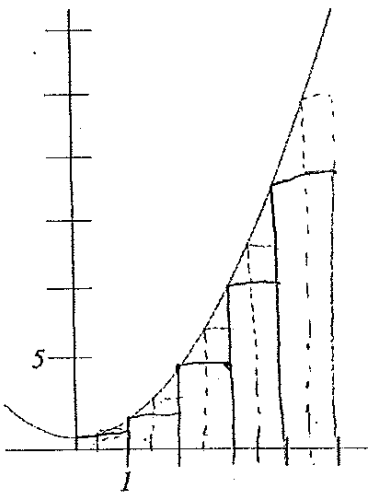
$$\begin{aligned} f(1/2) &= 1.25 \\ f(1\frac{1}{2}) &= 3.25 \\ f(2\frac{1}{2}) &= 7.25 \\ f(3\frac{1}{2}) &= 13.25 \\ f(4\frac{1}{2}) &= 21.25 \end{aligned}$$

$$\Delta x = 1/2$$

$$\sum_{i=1}^{10} f(x_i) \Delta x$$

$$\Rightarrow \frac{1}{2} (60 + 1.25 + 3.25 + 7.25 + 13.25 + 21.25) = \boxed{53.125}$$

Example 2: Do the same as example one only use left endpoints.



$$\begin{aligned} f(0) &= 1 \\ f(1) &= 2 \\ f(2) &= 5 \\ f(3) &= 10 \\ f(4) &= 17 \end{aligned}$$

$$\Delta x = 1$$

$$\sum_{i=1}^5 f(x_{i-1}) \Delta x$$

$$= 1(1 + 2 + 5 + 10 + 17) = \boxed{35}$$

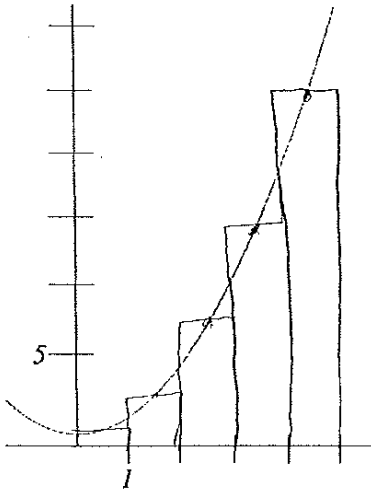
$$\begin{aligned} f(1/2) &= 1.25 \\ f(1\frac{1}{2}) &= 3.25 \\ f(2\frac{1}{2}) &= 7.25 \\ f(3\frac{1}{2}) &= 13.25 \\ f(4\frac{1}{2}) &= 21.25 \end{aligned}$$

$$\Delta x = 1/2$$

$$\sum_{i=1}^{10} f(x_{i-1}) \Delta x$$

$$= \frac{1}{2} (35 + 1.25 + 3.25 + 7.25 + 13.25 + 21.25) = \boxed{40.625}$$

Example 3: Do the same as example 1 only use midpoints and 5 rectangles.



midpoints are

$$f(\frac{1}{2}) = 1.25$$

$$f(1\frac{1}{2}) = 3.25$$

$$f(2\frac{1}{2}) = 7.25$$

$$f(3\frac{1}{2}) = 13.25$$

$$f(4\frac{1}{2}) = 21.25$$

$$\Delta x = 1$$

$$\sum_{i=1}^5 f(x_i^*) \Delta x$$

$$= 1(1.25 + 3.25 + 7.25 + 13.25 + 21.25)$$

$$= \boxed{46.25}$$

Between example 1 and 2 which is the over estimate and which is the underestimate? #1 over,

#2 under

Out of all 3 examples, which do you think is the best estimate?

Why? 3, it seems to be the average of the upper and lower estimates.

Distance = velocity x time

What happens if the velocity is not constant?

Example 4: Page 341 #11.

(1) lower  $\Rightarrow L_6 = .5(4.2 + 10.8 + 14.9 + 12.1 + 19.4)$   
 $= 34.7 \text{ ft}$

Homework: page 341 (1-5, 12)

upper  $\Rightarrow R_6 = .5(4.2 + 10.8 + 14.9 + 18.1 + 19.4 + 20.2)$   
 $= 44.8 \text{ ft}$