Name Answer Key
Period _____ Date _____

Calculus Review 5.1 to 5.2

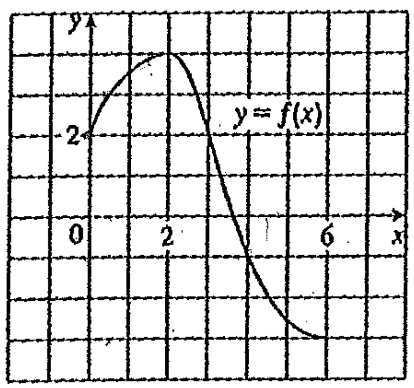
1. Use the given graph of f to find the Riemann sum with six subintervals. Take the sample points to be

   a. left endpoints $\Delta x = 1$

   $1(2+3.5+4+2+-1+-2.5)=8$

   b. midpoints $\Delta x = 1$

   $1(3+3.8+3.5+.5+-2+-2.8)=6$



$\frac{1}{2}$ $1\frac{1}{2}$ $2\frac{1}{2}$ $3\frac{1}{2}$ $4\frac{1}{2}$ $5\frac{1}{2}$

0, 1, 2, 3, 4, 5, 6

2, 3.5, 4, 2, -1, -2.5, -3

2. Given the function $f(x) = x^2 - x$ over the interval $0 \le x \le 2$

   a. Evaluate the sum using four subintervals and using right endpoints.

   $\Delta x = \frac{2-0}{4} = .5$ $\quad .5, 1, 1.5, 2$

   $.5(-.25, 0, .75, 2) = 1.25$

   b. Use the definition of the definite integral to evaluate $\int_0^2 (x^2 - x)\,dx$.

   $\Delta x = \frac{2-0}{n} = \frac{2}{n}$ $\quad x_i = \frac{2i}{n}$

   $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(\frac{2i}{n}\right)^2 - \frac{2i}{n}\right)\Delta x$

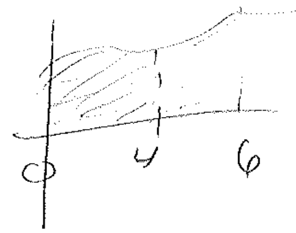
   $\frac{4 \cdot 8}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) - \frac{4}{n^2}\left(\frac{n(n+1)}{2}\right)$

   $\lim_{n\to\infty} \frac{4}{3}\left(\frac{n}{n}+\frac{1}{n}\right)\left(\frac{2n}{n}+\frac{1}{n}\right) - 2\left(\frac{n}{n}+\frac{1}{n}\right) = \frac{8}{3} - 2 = \frac{2}{3}$

3. Express $\lim_{n\to\infty}\sum_{i=1}^{n}\sin x_i \Delta x$ as a definite integral on the interval $[0,\pi]$.

$\int_0^{\pi} \sin x \, dx$

4. If $\int_0^6 f(x)dx = 10$ and $\int_0^4 f(x)dx = 7$, find $\int_4^6 f(x)dx$.



$10 - 7 = 3$

5. Write $\int_0^2 e^{3x}dx$ as a limit of Riemann sums taking sample points to be right endpoints. Do not evaluate!

$\Delta x = \frac{2-0}{n} = \frac{2}{n}$ $\quad x_i = \frac{2i}{n}$

$\lim_{n\to\infty} \sum_{i=1}^{n} e^{3(2i/n)} \Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \left(e^{\left(\frac{6i}{n}\right)}\left(\frac{2}{n}\right)\right)$

$= \lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} e^{\left(\frac{6i}{n}\right)}$

6. Use the midpoint rule with n=4 to approximate the integral $\int_1^5 x^2 e^{-x}dx$. Round your answer to four decimal places.

$\Delta x = \frac{5-1}{4} = 1$ $\quad$ x mpts $= 1.5, 2.5, 3.5, 4.5$

$1(.50204 + .51303 + .36992 + .22496) \approx 1.6099$

7. Evaluate the integral $\int_1^4 (x^2 + 2x - 5)dx$.

$\Delta x = \frac{4-1}{n} = \frac{3}{n}$ $\quad x_i = \frac{3i}{n} + 1$

$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(\frac{3i}{n}+1\right)^2 + 2\left(\frac{3i}{n}+1\right) - 5\right)\left(\frac{3}{n}\right)$

$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{9i^2}{n^2} + \frac{6i}{n} + 1 + \frac{6i}{n} + 2 - 5\right)\frac{3}{n}$

$\frac{27}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) + \frac{36}{n^2}\left(\frac{n(n+1)}{2}\right) - \frac{6n}{n}$

$\frac{9}{2}\left(\frac{n}{n}+\frac{1}{n}\right)\left(\frac{2n}{n}+\frac{1}{n}\right)$

$+ 18\left(\frac{n}{n}+\frac{1}{n}\right) - 6$

$9 + 18 - 6 = 21$

8. Express $\int_1^{10} (x - 4\ln x)dx$ as the limit of Riemann sums. Do not evaluate!

$\Delta x = \frac{10-1}{n} = \frac{9}{n}$ $\quad x_i = \frac{9i}{n} + 1$

$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{9i}{n} + 1 - 4\ln\left(\frac{9i}{n} + 1\right)\right)\frac{9}{n}$

9. Express $\int_2^{10} x^3 dx$ as the limit of Riemann sums and evaluate.

$\Delta x = \frac{8}{n}$ $\quad x_i = \frac{8i}{n} + 2$

$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{8i}{n} + 2\right)^3 \left(\frac{8}{n}\right)$

$\lim_{n\to\infty} \frac{8}{n} \sum_{i=1}^{n} \left(\frac{512 i^3}{n^3} + 3\left(\frac{8i}{n}\right)^2(2) + 3\left(\frac{8i}{n}\right)2^2 + 8\right)$

$\lim_{n\to\infty} \left(\frac{8}{n}\right)\frac{512}{n^3}\left(\frac{n^2(n+1)^2}{4}\right) + \frac{3072}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) + \frac{768}{n^2}\left(\frac{n(n+1)}{2}\right)$

$1024\left(\frac{n}{n} + \frac{1}{n}\right)^2 + 512(1)(2) + 384(1) + 64 = 2496$

10. What is $\int_{-2}^{2} f(x)dx + \int_{2}^{5} f(x)dx - \int_{-2}^{-1} f(x)dx$ as a single integral?

$\int_{-1}^{5} f(x)\,dx$