Section 4.8 Notes

Antiderivative: A function F is called an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I.

General form: $F(x) + C$ where C is an arbitrary constant.

$$x^n = \frac{x^{n+1}}{n+1}$$

Let's try to find some with an educated guess…

Example 1: Find the most general form of the antiderivative for each:

a) $f(x) = \sin x$

$F(x) = -\cos x + C$

b) $f(x) = 4x^3$

$F(x) = \frac{4x^4}{4} = x^4 + C$

Look at table of Antiderivatives.

Example 2: Find f if $f'(x) = e^x + \frac{4}{1+x^2}$ and $f(0) = -2$.

$f(x) = e^x + 4\arctan x + C$

$-2 = e^0 + 4\arctan(0) + C$

$-3 - 4\arctan 0 = C$

$-3 - 0 = C$

$-3 = C$

$\therefore \boxed{f(x) = e^x + 4\arctan x - 3}$

Example 3: Find f if $f''(x) = 12x^2 + 6x - 4$ and $f(0) = 4$ and $f(1) = 1$.

$f'(x) = \frac{12x^3}{3} + \frac{6x^2}{2} - 4x$

$f'(x) = 4x^3 + 3x^2 - 4x + C$

$4 = C$

$f'(x) = 4x^3 + 3x^2 - 4x + 4$

$f(x) = \frac{4x^4}{4} + \frac{3x^3}{3} - \frac{4x^2}{2} + 4x + D$

$f(x) = x^4 + x^3 - 2x^2 + 4x + D$

$1 = 1 + 1 - 2 + 4 + D$

$-3 = D$

$\boxed{\therefore f(x) = x^4 + x^3 - 2x^2 + 4x - 3}$

Example 4: A particle moves in a straight line and has acceleration given by $a(t) = 6t + 4$. Its initial velocity is $v(0) = -6$ cm/s and its initial displacement is $s(0) = 9$cm. Find its position function.

$v(t) = \frac{6t^2}{2} + 4t + C = 3t^2 + 4t + C$

$v(t) = 3t^2 + 4t - 6$

$s(t) = \frac{3t^3}{3} + \frac{4t^2}{2} - 6t + D$

$\boxed{s(t) = t^3 + 2t^2 - 6t + 9}$

Homework: page 321 (1-7, 11-13, 17-21, 24, 28-31, 34, 41, 42)

Table of Antidifferentiation

Function	Particular Differentiation
$Cf(x)$	$cF(x)$
$f(x) + g(x)$	$F(x) + G(x)$
$x^n \quad (n \neq -1)$	$\frac{x^{n+1}}{n+1}$
$1/x$	$\ln \lvert x \rvert$
e^x	e^x
$\cos x$	$\sin x$
$\sin x$	$-\cos x$
$\sec^2 x$	$\tan x$
$\sec x \tan x$	$\sec x$
$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x$
$\frac{1}{1+x^2}$	$\tan^{-1} x$