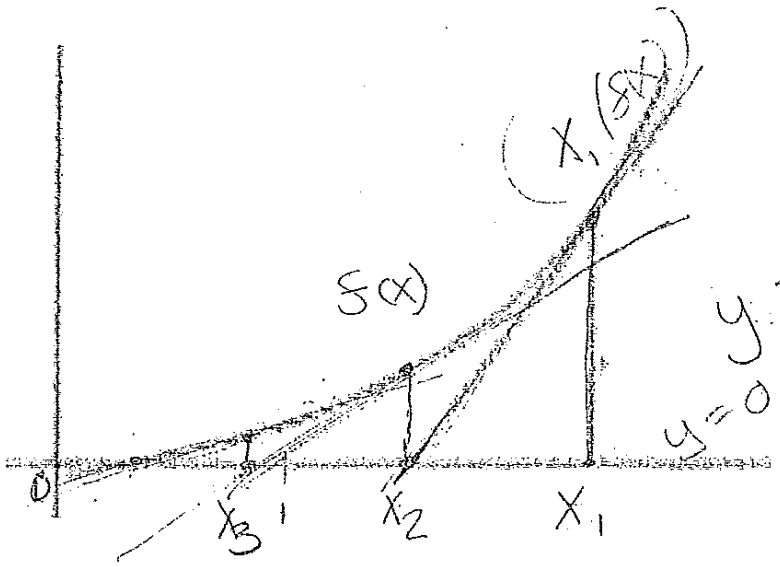
Calculus Sec 4.8 notes



$y - f(x_1) = f'(x_1)(x - x_1)$

$-f(x_1) = -f'(x_1)(x_1 - x)$

$\frac{-f(x_1)}{f'(x_1)} = x_1 - x$

$x_1 - \frac{f(x_1)}{f'(x_1)} = x$

Newton's Method: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

To compute $\sqrt{5}$. $\sqrt{5} \approx 2.23606797$

$\sqrt{5} = x \qquad 5 = x^2 \qquad 0 = x^2 - 5$

$x_{n+1} = x_n - \dfrac{x_n^2 - 5}{2x_n}$

$x_1 = 2$

$x_2 = 2 - \dfrac{2^2 - 5}{2 \cdot 2} = 2.25$

$x_3 = 2.25 - \dfrac{2.25^2 - 5}{2 \cdot 2.25} \approx 2.2361$

$x_5 = 2.23606797$

Example 2: Starting with $x_1 = 2$ find the third approximation x_3 to the root of the equation $x^3 - 2x - 5 = 0$.

$$X_2 = X_1 - \frac{X_1^3 - 2X_1 - 5}{3X_1^2 - 2}$$

$$X_2 = 2 - \frac{2^3 - 2\cdot 2 - 5}{3\cdot 2^2 - 2} = 2.1$$

$$X_3 = 2.1 - \frac{(2.1)^3 - 2\cdot 2.1 - 5}{3\cdot 2.1^2 - 2} = 2.1 - \frac{.061}{11.23} = 2.094568\overline{12}$$

Example 3: Use Newton's Method to find $\sqrt[6]{2}$ correct to eight decimal places.

$x^6 - 2 = 0$

$x_1 = 1$ $\qquad x_{n+1} = x_n - \dfrac{x_n^6 - 2}{6x_n^5}$

$x_2 \approx 1.1\overline{6}$

$x_3 \approx 1.12644368$

$x_4 \approx 1.12249707$

$x_5 \approx 1.12246205$) ok to 8 dec spots

$x_6 \approx 1.12246205$

Homework: page 325 #1, 4, 5-9, 11