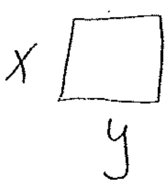
Section 4.6

First derivative test for absolute extreme values:
Suppose that c is a critical number of a continuous function f defined on an interval.

a. If $f'(x) > 0$ for all $x < c$ and $f'(x) < 0$ for all $x > c$, then $f(c)$ is the absolute max.

b. If $f'(x) < 0$ for all $x < c$ and $f'(x) > 0$ for all $x > c$, then $f(c)$ is the absolute min.

Example1: Find the dimensions of a rectangle with area 625 m² whose perimeter is as small as possible.



$A(x) = xy$

$625 = xy$

$y = \frac{625}{x}$

$P(x) = 2x + 2y$

$P(x) = 2x + \frac{1250}{x}$

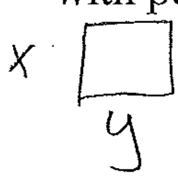
$P'(x) = 2 - \frac{1250}{x^2}$

$P'(x) < 0 \qquad 2 - \frac{1250}{x^2} < 0 \qquad 2 < \frac{1250}{x^2} \qquad x^2 < 625$

$|x| < 25$

$S'(x) < 0$ if $|x| < 25$ $\quad \therefore \boxed{x = 25\text{ m},\ y = 25\text{ m}}$

Example 2: Find the dimensions and area of a rectangle with perimeter 324 cm whose area is as large as possible.



$2x + 2y = 324$

$y = 162 - x$

$S'(x) = 162 - 2x > 0$

$162 > 2x$

$81 > x$

$S(x) = xy$

$S(x) = x(162 - x) = 162x - x^2$

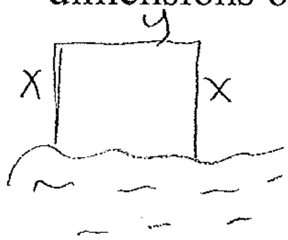
$S'(x) > 0$ for $x < 81$

$\therefore x = 81$ abs max

81×81

$\boxed{A = 6561\text{ cm}^2}$

Example 3: A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. No fence is needed along the river. What are the dimensions of the field that has the largest area?



$P(x) = 2x + y$

$2400 = 2x + y$

$y = 2400 - 2x$

$A(x) = xy$

$A(x) = x(2400 - 2x)$

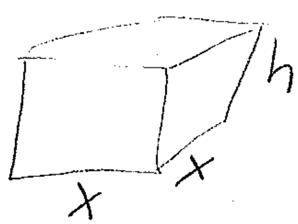
$A(x) = 2400x - 2x^2$

$A'(x) = 2400 - 4x = 0$

$x = 600 \text{ ft}$

$\therefore \; 600\text{ft} \times 1200\text{ft} = 720{,}000\text{ft}^2$

Example 4: If 600 cm² of material is available to make a box with a square base and an open top, find the largest possible volume of the box.



$A(x) = x^2 + 4xh$

$600 = x^2 + 4xh$

$h = \dfrac{600 - x^2}{4x}$

$V(x) = x^2 h$

$V(x) = x^2\left(\dfrac{600 - x^2}{4x}\right)$

$V(x) = 150x - \frac{1}{4}x^3$

$V'(x) = 150 - \frac{3}{4}x^2 = 0$

$150 = \frac{3}{4}x^2$

$200 = x^2$

$x = 10\sqrt{2}$

$V(10\sqrt{2}) = 150(10\sqrt{2}) - \frac{1}{4}(10\sqrt{2})^3$

$\boxed{\approx 1414 \text{cm}^3}$

Homework: page 305 (2-6, 9-14)

Calculus Section 4.6 day 2

Cost function C(x): a function of producing x units of a certain product. $C(x)$

Average Cost function c(x): C(x)/x, cost per unit. $c(x)$

Marginal Cost function : rate of change of C with respect to x. (C') $\frac{dC}{dx}$

Minimum average cost: marginal cost = average cost
$C' = c(x)$

Example 1: A company estimates that the cost (in dollars)of producing x items is
$C(x) = 2600 + 2x + 0.001x^2$.

a. Find the cost, average cost, and marginal cost of producing 1000 items.

$C(1000) = 2600 + 2(1000) + 0.001(1000)^2 = \$5600$

$c(1000) = 5600/1000 = \$5.60/\text{unit}$

$C'(x) = 2 + .002x$

$C'(1000) = 2 + .002(1000) = \$4.00/\text{unit}$

$2+0.002x = \frac{2600}{x} + 2 + .001x$

$0.001x^2 = 2600$ $\quad x \approx 1612$ $\quad C(1612) = \$5.22/\text{item}$

$x^2 = 2,600,000$

b. At what production level will the average cost be lowest, and what is this minimum cost? $C' = c(x)$

Demand function = price function p(x): price per unit that the company can charge if it sells x units.

Revenue function = sales function: $R(x) = xp(x)$, total revenue for x items sold.

Marginal Revenue function: R', rate of change of revenue with respect to x. $\frac{dR}{dt}$

Profit Function: $P(x) = R(x) - C(x)$

Marginal Profit function: P', rate of change of profit with respect to x. $\frac{dP}{dx}$

Maximum Profit: marginal revenue = marginal cost

$$R' = C'$$

Example 2: Determine the production level that will maximize the profit for a company with cost and demand functions: $C(x) = 84 + 1.26x - 0.01x^2 + 0.00007x^3$ and $p(x) = 3.5 - 0.01x$

$R'(x) = C'(x)$

$R(x) = xp(x) = 3.5x - 0.01x^2$

$C'(x) = 1.26 - 0.02x + 0.00021x^2$

$R'(x) = 3.5 - 0.02x$

$3.5 - 0.02x = 1.26 - 0.02x + 0.00021x^2$

$2.24 = 0.00021x^2$

$10666\frac{2}{3} = x^2$

$\boxed{103 \approx x}$

Example 3: BHS wants to get new uniforms for basketball. They find that the average bb attendance is 400 fans when the ticket price is $5. When they lower the price to $4, the average bb attendance rises to 550 fans.

a. Find the demand function, assuming it is linear.

$y-y=m(x-x)$ $(400,5)$ $(550,4)$

$\frac{4-5}{550-400} = \frac{-1}{150} = m$

$y-5 = \frac{-1}{150}(x-400)$

$y-5 = \frac{-x}{150} + 2\frac{2}{3}$

$\boxed{p(x) = \frac{-x}{150} + 7\frac{2}{3}}$

b. How should ticket prices be set to maximize revenue?

$R(x) = xp(x)$

$R(x) = \frac{-x^2}{150} + 7\frac{2}{3}x$

$R'(x) = \frac{-x}{75} + 7\frac{2}{3}$

$x = 575$

$p(575) = \frac{-575}{150} + 7\frac{2}{3} = \$3.83/\text{seat}$

Homework: page 308 (43-47)