Sec 4.6 day 1 p. 305 (2-6, 9-14)

(2) x $\qquad$ $x(x+100)$ is minimum

$(x+100)$

$f(x) = x^2 + 100x$

$f'(x) = 2x + 100$

$x = -50$ $\qquad$ $\boxed{\therefore 50, -50}$

(3) $xy = 100$ $\qquad$ $x+y$ is min

$y = \frac{100}{x}$ $\qquad$ $f(x) = x + 100x^{-1}$

$f'(x) = 1 - \frac{100}{x^2} = 0$ $\qquad$ $x^2 = 100$

$x = \pm 10$

$$\boxed{\begin{array}{l} \therefore x = 10 \\ y = 10 \end{array}}$$

(4) $x+y = 16$ $\qquad$ $y = 16 - x$

$x^2 + y^2$ is min

$f(x) = x^2 + (16-x)^2$

$f'(x) = 2x - 2(16-x) = 0$

$f'(x) = 4x - 32$

$\therefore x = 8$ $\qquad$ $y = 8$ $\qquad$ $\boxed{8^2 + 8^2 = 128}$

(5) $2x + 2y = 100$ $\qquad A = xy$

$y = 50 - x$ $\qquad f(x) = 50x - x^2$

$f'(x) = 50 - 2x$

$x = 25$ $\qquad \boxed{\therefore x = y = 25m}$

(6) $xy = 1000$ $\qquad 2x + 2y = P$

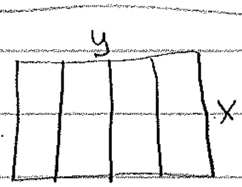
$y = \frac{1000}{x}$ $\qquad P(x) = 2x + \frac{2000}{x}$

$P'(x) = 2 - \frac{2000}{x^2} = 0$ $\qquad x^2 = 1000$

$x = 10\sqrt{10}$

$\boxed{\therefore 10\sqrt{10}m \times 10\sqrt{10}m}$

(9) ab)



cd) $5x + 2y = 750$ $\qquad y = -\frac{5}{2}x + 375$

max $xy = A$

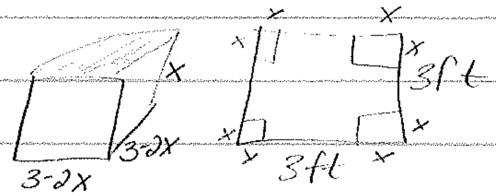
e) $A(x) = -\frac{5}{2}x^2 + 375x$

f) $-5x + 375 = A'(x) = 0$

$x = 75$ $\qquad \boxed{x = 75ft \quad y = 187.5ft}$

max area of $14,062.5ft^2$

(10) ab)



c-e) $V(x) = (3-2x)^2 x$

$V'(x) = -4(3-2x)x + (3-2x)^2$

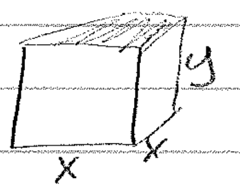
$V'(x) = (3-2x)(-4x+3-2x)$

$V'(x) = (3-2x)(-6x+3)$

$x = 3/2$ or $x = 1/2$

$V(\frac{3}{2}) = 0 \qquad \therefore V(1/2) = 2 \cdot 2 \cdot \frac{1}{2} = \boxed{2\text{ ft}^3}$

(11)



$SA = 1200$

$SA(x,y) = x^2 + 4xy$

$1200 = x^2 + 4xy \Rightarrow y = \frac{1200 - x^2}{4x} = \frac{300}{x} - \frac{x}{4}$

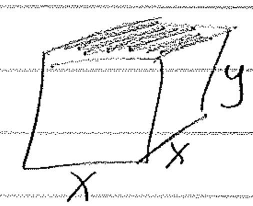
$V(x) = x^2 y = 300x - \frac{x^3}{4}$

$V'(x) = 300 - \frac{3x^2}{4} = 0 \qquad \frac{3}{4}x^2 = 300 \qquad x^2 = 400$

$x = 20$

$\boxed{x = 20 \quad y = 10 \quad \text{largest volume is } 20^2(10) = 4000\text{ cm}^3}$

(12)



$x^2 y = 32000 \qquad SA = x^2 + 4xy$

$y = \frac{32000}{x^2} \qquad SA(x) = x^2 + \frac{128,000}{x}$

$SA'(x) = 2x - \frac{128,000}{x^2} = 0 \qquad x^3 = 64000 \qquad x = 40$

$\boxed{\therefore x = 40,\ y = 20 \Rightarrow 40 \times 40 \times 20\text{ cm}}$

(13) a) $xy = A$ $\quad P = 2x + 2y$

$y = \frac{A}{x}$ $\quad P = 2x + \frac{2A}{x}$

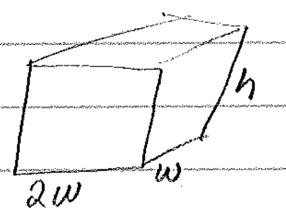
$2 - \frac{2A}{x^2} = P'(x) = 0$ $\quad 2x^2 = 2A$

$x^2 = A$ minimizes perimeter

b) $2x + 2y = P$ $\quad A = xy$

$y = \frac{P}{2} - x$ $\quad A(x) = \frac{Px}{2} - x^2$

$A'(x) = \frac{P}{2} - 2x = 0$ $\quad P = 4x$ maximizes area

(14) 

$V = 10\ m^3$ $\quad SA = 2w^2 + (2wh)2 + 2(wh)$

$V = 2w^2 h$ $\quad SA = 2w^2 + 6wh$

$10 = 2w^2 h$ $\quad SA(w) = 2w^2 + \frac{30}{w}$

$h = \frac{5}{w^2}$

$C(w) = 10(2w^2) + 6\left(\frac{30}{w}\right)$

$C(w) = 20w^2 + \frac{180}{w}$

$C'(w) = 40w - \frac{180}{w^2} = 0$ $\quad w^3 = \frac{9}{2}$ $\quad w = \sqrt[3]{9/2}$

$\therefore C\left(\sqrt[3]{9/2}\right) = 20\left(\sqrt[3]{9/2}\right)^2 + \frac{180}{\sqrt[3]{9/2}} \approx \boxed{\$\ 163.54}$

Sec 4.6 day 2 page 308 (43-47)

(43) a) $c'(x) = \dfrac{C'(x)x - C(x)}{x^2}$

$c'(x) = 0$ when $xC'(x) - C(x) = 0$

or when $C'(x) = \dfrac{C(x)}{x} = c(x)$

$\therefore$ marg = avc

b) i) $C(1000) = \$342,491$

$\dfrac{C(1000)}{1000} = \$342.49$ /unit

$C'(x) = 200 + 6x^{1/2}$

$C'(1000) = \$389.74$ /unit

ii) $\dfrac{16,000 + 200x + 4x^{3/2}}{x} = 200 + 6x^{1/2}$

$16,000 + \cancel{200x} + 4x^{3/2} = \cancel{200x} + 6x^{3/2}$

$2x^{3/2} = 16000$

$x^{3/2} = 8000$

$x = \sqrt[3]{(8000)^2}$ $\boxed{= 400}$

iii) $c(400) = \$320$ /unit

44 a) $P(x) = R(x) - C(x)$

$P'(x) = R'(x) - C'(x) = 0$

$R'(x) = C'(x)$

b) $R(x) = x\,p(x) = 1700x - 7x^2$

$R'(x) = 1700 - 14x$

$C'(x) = 500 - 3.2x + 0.012x^2$

$R'(x) = C'(x)$

$0 = 1200 - 10.8x - .012x^2$

poly smlt $\boxed{x = 100}$

45 a) $(27{,}000, 10)$ $\frac{2}{-6000} = \frac{-1}{3000}$

$(33{,}000, 8)$

$y - 8 = \frac{-1}{3000}(x - 33{,}000)$

$y = \frac{-x}{3000} + 19 = p(x)$

a) $R(x) = x\,p(x) = \frac{-x^2}{3000} + 19x$

b) $R'(x) = \frac{-x}{1500} + 19$

$x = 28{,}500 \Rightarrow p(28{,}500) = \boxed{\$9.50}$

(46) $(20, 10)$ $\quad m = \frac{-1}{2} \quad y - 10 = -\frac{1}{2}(x - 20)$
$(18, 11)$

a) $y = -\frac{1}{2}x + 20 = p(x)$

b) $R(x) = x\,p(x) = -\frac{x^2}{2} + 20x$

$R'(x) = -x + 20$
$C(x) = 6x$
$C'(x) = 6$

$-x + 20 = 6$
$-x = -14$
$x = 14$

$p(14) = \frac{-14}{2} + 20 = \$13$

(47) a) $(1000, 450)$ $\quad m = \frac{-10}{100} = \frac{-1}{10} \quad y - 450 = -\frac{1}{10}(x - 1000)$
$(1100, 440)$

a) $y = -\frac{x}{10} + 550 = p(x)$

b) $R(x) = -\frac{x^2}{10} + 550x \qquad R'(x) = -\frac{1}{5}x + 550 = 0$
$x = 2750$

$p(2750) = 275$ so the rebate should be $450 - 275 = \$175$

c) $R' = C' \Rightarrow -\frac{1}{5}x + 550 = 150$
$x = 2000 \Rightarrow p(2000) = \$350 \quad \therefore 450 - 350 = \$100$ rebate