Name Answer Key
Period _____ Date 2/12/2013

Calculus Section 4.6-4.9 Review

Use Calculus to answer the following questions.

1. A Hockey team plays in an arena with a seating capacity of 15,000 spectators. With the ticket price set at $12, average attendance at a game has been 11,000. A market survey indicates that attendance would be 12,500 if the ticket price is $11. Assuming the demand function is linear, what should they charge to maximize their Revenue?

$(11{,}000, 12)\ (12{,}500, 11)$ $\quad p(x) - 12 = -\frac{1}{1500}(x - 11{,}000)$

$\frac{12-11}{11000-12500} = -\frac{1}{1500}$ $\quad p(x) = -\frac{1}{1500}x + 19\frac{1}{3}$

$R(x) = -\frac{x^2}{1500} + 19\frac{1}{3}x$ $\quad R'(x) = -\frac{x}{750} + 19\frac{1}{3}$ $\quad R'(x) = 0$ when $x \approx 14500$

$\boxed{p(14500) = \$9.67/\text{seat}}$

2. A manufacturer determines that the cost of making x items is $C(x) = 1800 + 25x - 0.2x^2 + 0.001x^3$ and the demand function is $p(x) = 48.2 - 0.03x$. Find the production level for maximum profit.

$R' = C' \Rightarrow R(x) = x \cdot p(x) = 48.2x - .03x^2$

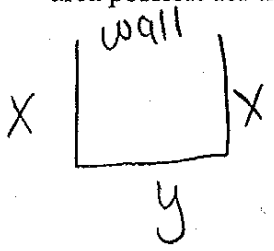
$R' = 48.2 - .06x$ $\quad C' = 25 - .4x + .003x^2$

$48.2 - .06x = 25 - .4x + .003x^2$

$23.2 + .34x - .003x^2 = 0$

$x \approx 141.28$ $\quad$ DO (141)

3. Kelli is going to start a day care for the summer and has 10 feet of gates she can use to fence in the kids while her day time soaps are on. She wants to make a rectangular play pin but is able to use the wall as one of the sides. In square feet, what is the maximum area possible for this pen?



$A(x) = xy$ $\quad P(x) = 2x + y \Rightarrow 10 = 2x + y$

$\therefore y = 10 - 2x$

$A(x) = 10x - 2x^2$

$A'(x) = 10 - 4x = 0$ $\quad x = 2.5$

$A(2.5) = \boxed{12.5\ ft^2}$

4. Find two positive integers such that the sum of the first number and four times the second number is 1000 and the product of the numbers is as large as possible.

$x + 4y = 1000$ $\quad xy = f(x)$

$y = -\frac{1}{4}x + 250$ $\quad -\frac{1}{4}x^2 + 250x = f(x)$

$f'(x) = -\frac{1}{2}x + 250 = 0$

$\boxed{x = 500 \quad y = 125}$

5. A company has cost function $C(x) = 5000 + 0.003x + 0.01x^2 + 5x^3$. Find the average cost function and marginal cost function. At what production level will the average cost be minimized? What is that average cost?

$c(x) = 5000 + 0.003x + .01x^2 + 5x^3$

$C'(x) = .003 + .02x + 15x^2$

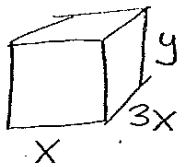
$c(x) = C'(x) \Rightarrow 5000 + .003x + .01x^2 + 5x^3 = .003x + .02x^2 + 15x^3$

$5000 - .01x^2 - 10x^3 = 0$

$x \approx 7.9$

$\boxed{c(7.9) = \$945.04}$

6. A rectangular storage container with an open top is to have a volume of 12 centimeters cubed. The length of the base is 3 times the length of the width. Material for the base is \$.90 a square centimeter and material for the sides is \$.50 a square centimeter. Find the cost of materials for the cheapest such container.



$V(x) = 3x^2y$

$12 = 3x^2y$

$y = \frac{4}{x^2}$

$SA(x) = 3x^2 + 8xy$

$SA(x) = 3x^2 + \frac{32}{x}$

$C(x) = .9(3x^2) + .5\left(\frac{32}{x}\right)$

$C(x) = 2.7x^2 + \frac{16}{x}$

$C'(x) = 5.4x - \frac{16}{x^2} = 0$

$5.4x^3 = 16$

$x^3 = 2.962962962$

$x = 1.436629$

$\boxed{C(1.436629) \approx \$16.71}$

8. Find the most general form of $f(x)$ if $f''(x) = -\cos x + 2 + 20x^3$.

$f'(x) = -\sin x + 2x + 5x^4 + C$

$f(x) = \cos x + x^2 + x^5 + Cx + D$

On 9-10, find $f(x)$.

9. $f'(x) = \frac{2}{1+x^2}$ $f(0) = -1$

$f(x) = 2\arctan x + C$

$C = -1$

$\boxed{f(x) = 2\arctan x - 1}$

10. $f''(x) = 1 - 6x + 48x^2$

$f(0) = 1$ and $f'(0) = 2$

$f'(x) = x - 3x^2 + 16x^3 + 2$

$\boxed{f(x) = \frac{1}{2}x^2 - x^3 + 4x^4 + 2x + 1}$

11. A particle moves with acceleration function $a(t) = 6t - 4$. Its initial velocity is $v(0) = 3$ m/s and its displacement is $s(0) = 4$ m. Find its position function after t seconds.

$v(t) = 3t^2 - 4x + 3$

$s(t) = t^3 - 2t^2 + 3t + 4$

12. A particle moves along a straight line with velocity function $v(t) = \cos t - \sin t$ and its initial displacement is $s(0) = 1$. Find its position function $s(t)$.

$s(t) = \sin t + \cos t + C$

$s(0) = 0 + 1 + C = 1$

$C = 0$

$\boxed{\therefore s(t) = \sin t + \cos t}$