Section 4.5 Notes

Lapital

Recall from chapter 2:

Definition 7: If n is a positive integer, then

$$\lim_{x\to\infty}\frac{1}{x^n}=0 \qquad \lim_{x\to-\infty}\frac{1}{x^n}=0$$

Definition 8: $\lim_{x\to-\infty} e^x = 0$

Direct substitution: Find $\lim_{x\to 2}\left(x^2-x\right)$.

$2^2-2=2$

Find $\lim_{x\to\infty}\left(x^2-x\right)$.

$\infty^2-\infty$

$\infty-\infty=?$

We have indeterminate forms when using direct substitution. They are:

$\frac{0}{0}$ $\qquad \frac{\infty}{\infty}$ $\qquad 0\bullet\infty$ $\qquad \infty-\infty$

0^0 $\qquad \infty^0$ $\qquad 1^\infty$

L'Hospital's Rule: Suppose f and g are differentiable and $g'(x)$ does not equal 0 near a (except possibly at a). Suppose that

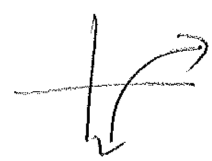
$$\lim_{x\to a} f(x) = 0 \quad \text{and} \quad \lim_{x\to a} g(x) = 0$$

or that

$$\lim_{x\to a} f(x) = \pm\infty \quad \text{and} \quad \lim_{x\to a} g(x) = \pm\infty$$

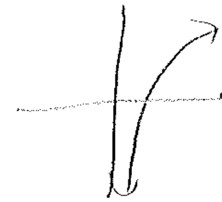
then $\lim\limits_{x\to a} \dfrac{f(x)}{g(x)} = \lim\limits_{x\to a} \dfrac{f'(x)}{g'(x)}$ if the limit on the right side exists or is + or – infinity.

Example 1: Find $\lim\limits_{x\to 1} \dfrac{\ln x}{x-1}$. $= \dfrac{0}{0}$



$$\overset{H}{=} \frac{\frac{1}{x}}{1} = 1$$

Example 2: Calculate $\lim_{x \to \infty} \frac{e^x}{x^2} = \frac{e^{\infty}}{\infty^2} = \frac{\infty}{\infty}$



$$\overset{H}{=} \frac{e^x}{2x} = \frac{e^{\infty}}{2\infty} = \frac{\infty}{\infty}$$

$$\overset{H}{=} \frac{e^{\infty}}{2} = \frac{\infty}{2} = \infty$$

Example 3: Evaluate $\lim_{x \to 0^+} x \ln x = 0 \cdot -\infty$

$$\lim_{x \to 0^+} \frac{\ln x}{x^{-1}} \overset{H}{=} \frac{\frac{1}{x}}{-x^{-2}}$$

$$= \frac{1}{x}\left(\frac{-x^2}{1}\right) = -x = 0$$

Example 4: Find $\lim_{x \to 0^+} x^x = 0^0$

$y = x^x$

$\lim_{x \to 0^+} y = \lim_{x \to 0^+} x^x$

$\lim_{x \to 0^+} \ln y = \lim_{x \to 0^+} x \ln x$

$\lim_{x \to 0^+} x \ln x \overset{=0}{\text{see ex 3}}$

$\lim_{x \to 0^+} \ln y = 0$

$\lim_{x \to 0^+} e^{\ln y} = e^0$

$\lim_{x \to 0^+} y = 1$

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