Sec 4.5 page 296 (1-8, 10-14, 19, 21-23, 29, 31, 39)

(1) a) $\frac{0}{0}$ b) $\frac{0}{\infty} = 0$ c) $\frac{1}{\infty} = 0$

d) $\frac{\infty}{0} = \pm\infty$ e) $\frac{\infty}{\infty}$

(2) a) $0 \cdot \infty$ b) $1 \cdot \infty = \infty$ c) $\infty \cdot \infty = \infty$

(3) a) $0 - \infty = -\infty$ b) $\infty - \infty$ c) $\infty + \infty = \infty$

(4) a) 0^0 b) 0 c) 1^∞ d) ∞^0

e) $\infty^\infty = \infty$ f) $\infty^{1/\infty} = \infty^0$

(4b) $[f(x)]^{p(x)} = \ln y = p(x) \ln f(x)$

$\lim_{x \to a} y = \lim_{x \to a} e^{\ln y} = 0.$

(5) $\lim_{x \to 1} \frac{x^2 - 1}{x^2 - x} = \frac{0}{0} \overset{H}{=} \frac{2x}{2x - 1} = 2$

(6) $\lim_{x \to 1} \frac{x^a - 1}{x^b - 1} = \frac{0}{0} \overset{H}{=} \frac{ax^{a-1}}{bx^{b-1}} = \frac{a}{b}$

(7) $\lim_{x \to (\pi/2)^+} \frac{\cos x}{1 - \sin x} = \frac{0}{0} \overset{H}{=} \frac{-\sin x}{-\cos x} = \tan x = +\infty$

(8) $\lim_{x\to 0} \frac{\sin 4x}{\tan 5x} = \frac{0}{0} \overset{H}{=} \frac{4\cos 4x}{5\sec^2 5x} = \boxed{\frac{4}{5}}$

(10) $\lim_{t\to 0} \frac{e^{3t}-1}{t} = \frac{0}{0} \overset{H}{=} \frac{3e^{3t}}{1} = \boxed{3}$

(11) $\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}} = \frac{\infty}{\infty} \overset{H}{=} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \frac{2}{\sqrt{x}} = \boxed{0}$

(12) $\lim_{\theta\to \pi/2} \frac{1-\sin\theta}{\csc\theta} = \frac{0}{1} = \boxed{0}$

(13) $\lim_{x\to 0^+} \frac{\ln x}{x} = \frac{-\infty}{0} = \boxed{-\infty}$

(14) $\lim_{x\to\infty} \frac{(\ln x)^2}{x} = \frac{\infty}{\infty} \overset{H}{=} \frac{2\ln x}{x} = \frac{\infty}{\infty} \overset{H}{=} \frac{2}{x} = \boxed{0}$

(19) $\lim_{x\to 0} \frac{e^x-1-x}{x^2} = \frac{0}{0} \overset{H}{=} \frac{e^x-1}{2x} = \frac{0}{0} \overset{H}{=} \frac{e^x}{2} = \boxed{\frac{1}{2}}$

(21) $\lim_{x\to 1} \frac{1-x+\ln x}{1+\cos \pi x} = \frac{0}{0} \overset{H}{=} \frac{-1+\frac{1}{x}}{-\pi \sin \pi x} = \frac{0}{0}$

$\overset{H}{=} \frac{-1}{-x^2 \pi^2 \cos \pi x} = \boxed{\frac{-1}{\pi^2}}$

(23) $\lim_{x\to 1} \frac{x^9 - 9x + a - 1}{(x-1)^2} = \frac{0}{0} \overset{H}{=} \frac{ax^{9-1} - a}{2(x-1)} = \frac{0}{0}$

$\overset{H}{=} \frac{a(a-1)x^{a-2}}{2} = \boxed{\frac{a(a-1)}{2}}$

(29) $\lim_{x\to 0} \cot 2x \sin 6x = \infty \cdot 0$

$\lim_{x\to 0} \frac{\sin 6x}{\tan 2x} = \frac{6\cos 6x}{2\sec^2 2x} = \frac{6(1)}{2(1)^2} = \boxed{3}$

(31) $\lim_{x\to\infty} x^3 e^{-x^2} = \infty \cdot 0 = \lim_{x\to\infty} \frac{x^3}{e^{x^2}} \overset{H}{=} \frac{3x^2}{2e^{x^2}}$

$= \frac{\infty}{\infty} \overset{H}{=} \frac{6x}{4e^{x^2}} = \frac{\infty}{\infty} \overset{H}{=} \frac{6}{8e^{x^2}} = \frac{6}{\infty} = \boxed{0}$

(39) $\lim_{x\to 0^+} x^{x^2} \Rightarrow \lim_{x\to 0^+} y = \lim_{x\to 0^+} x^{x^2}$

$\Rightarrow \lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} x^2 \ln x \Rightarrow 0 \cdot -\infty \quad \frac{\ln x}{x^{-1}} \overset{H}{=} \frac{\frac{1}{x}}{-x^{-2}} = -x$

$\lim_{x\to 0^+} \ln y = 0$

$\lim_{x\to 0^+} y = e^0 = \boxed{1}$