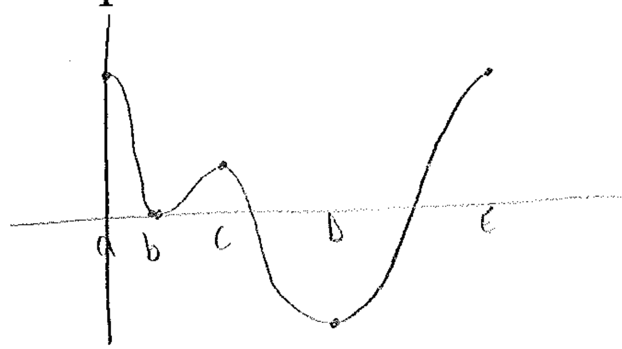
Section 4.2 Notes

Local maximum: at c if $f(c) \geq f(x)$ cannot occur at end point & must be defined

Local minimum: at c if $f(c) \leq f(x)$

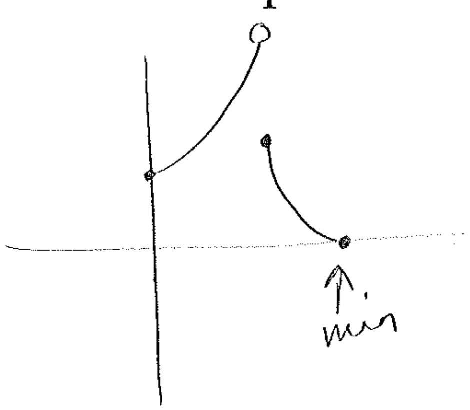
Absolute maximum or minimum: the greatest or lowest value of a function over entire function. (Does not include infinity).

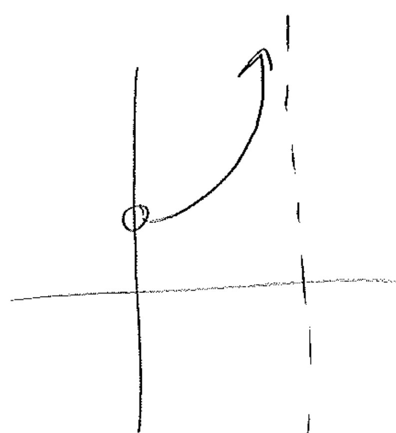
Example:



L max c
L min b, d
A max a & E
A min D

Non Examples:

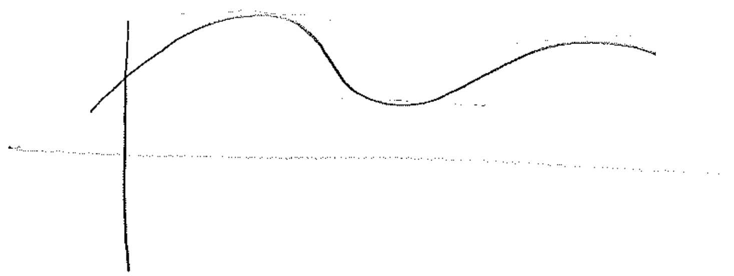




Neither

Fermat's Theorem: If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$. (Converse is not always true!!!!!)

Example:



Critical number: **1.** A number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist. **2.** If f has a local max or min at c, then c is a critical number of f.

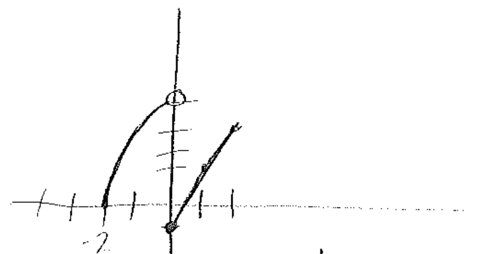
To find abs max or min over closed interval [a, b]:

1. Find the values of f at the critical numbers of f in (a, b).
2. Find the values of f at the endpoints of the interval.
3. The largest value from 1 and 2 is abs max and the smallest is abs min.

Example 3: Sketch

$$f(x)=\begin{cases}4-x^2 & \text{if } -2\le x \lhd 0\\ 2x-1 & \text{if } 0\le x\le 2\end{cases}$$

by hand and find abs max and abs min.



no abs max

absolute min -1

Example 4: Find the critical numbers of $s(t)=3t^4+4t^3-6t^2$.

$12t^3+12t^2-12t$

$12t\,(t^2+t-1)$ can't factor w/ out quad

$t=0$ so

or $\dfrac{-1\pm\sqrt{1-4(1)(-1)}}{2(1)}=\dfrac{-1\pm\sqrt{5}}{2}$

so $t=0,\ \dfrac{-1\pm\sqrt{5}}{2}$

Example 5: Find the abs max and abs min of

$f(t) = t\sqrt{4-t^2}$ over the interval $[-1, 2]$

$$f'(t) = \sqrt{4-t^2} + \frac{t(-\cancel{2}t)}{\cancel{2}\sqrt{4-t^2}} \qquad \text{LCD } \sqrt{4-t^2}$$

$$f'(t) = \frac{(4-t^2) - t^2}{\sqrt{4-t^2}} = \frac{4-2t^2}{\sqrt{4-t^2}}$$

numerator

$$2(2-t^2) = 0$$
$$4 - 2t^2 = 0$$
$$\sqrt{t^2} = \sqrt{2}$$
$$t = \pm\sqrt{2}$$
$$+\sqrt{2}$$

den

$$4 - t^2 \geq 0$$
$$4 \geq t^2$$
$$\pm 2 \geq t$$
$$-2 \leq t \leq 2$$
$$2$$

$[-1, 2]$

$f(-1) = -\sqrt{3}$ abs min

$f(\sqrt{2}) = 2$ abs max

$f(2) = 0$