Name _____
Period _____ Date_____

Calculus Notes for Section 4.1

Strategy on solving related rates problems:
1. Read problem carefully.
2. Draw and label a diagram if possible.
3. List what you know and represent it using derivative notation with respect to time.
4. List what you need to find as a derivative with respect to time.
5. Write a non specific equation.
6. Simplify the equation if necessary.
7. Differentiate both sides.
8. Substitute known values and simplify.

Example 1: If $x^2 + y^2 = 25$ and $dx/dt = 6$, find dy/dt when $x = 3$.

$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 0$

$3^2 + y^2 = 25$

$y^2 = 16$

$2(3)(6) + 2(4)\frac{dy}{dt} = 0$

$36 + 8\frac{dy}{dt} = 0$

$\frac{dx}{dt} = 6$

$\frac{dy}{dt} = ?$

$x = 3$

$y = 4$

$\frac{dy}{dt} = -4.5$

Example 2: Each side of a square is increasing at a rate of 6 cm/s. At what rate is the area of the square increasing when the area of the square is 16 square centimeters?

$A = x^2$

$\frac{dA}{dt} = 2x\frac{dx}{dt}$

$\frac{dA}{dt} = 2(4)(6)$

$\frac{dA}{dt} = 48\ cm^2/s$

$\frac{dx}{dt} = 6\ cm/s$

$\frac{dA}{dt} = ?$

$A = 16\ cm^2$

$x = 4\ cm$

$5x^2 = 6x$

Example 3: Air is being pumped into a spherical balloon so that its volume increases at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$d = 50cm$

$r = 25cm$

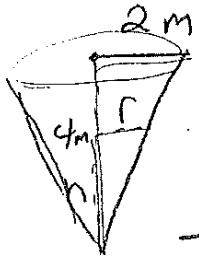
$\frac{dV}{dt} = 100$

$\frac{dr}{dt} = ?$

$100 = 4\pi(25)^2 \frac{dr}{dt}$

$\frac{1}{25\pi}\ cm/s = \frac{dr}{dt}$

Example 4: A water tank has the shape of an inverted circular cone with base radius 2 m and height 4 m. If water is being pumped into the tank at a rate of 2 m^3/min, find the rate at which the water level is rising when the water is 3 m deep.



$V = \frac{1}{3}\pi r^2 h$ $\frac{dV}{dt} = 2m^3/min$

$\frac{2}{4} = \frac{r}{h}$

$\frac{dh}{dt} = ?$

$h = 3m$

$r = \frac{h}{2}$

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$V = \frac{\pi h^3}{12}$

$\frac{dV}{dt} = \frac{\pi h^2}{4}\frac{dh}{dt}$

$2 = \frac{\pi 3^2}{4}\frac{dh}{dt}$

$\frac{8}{9\pi}\ m/min = \frac{dh}{dt}$ $\approx .28$

Homework: page 267 # 1, 2, 4, 5, 7-10, 16, 18, 27, 29, 33a.