Section 3.7 Notes

Four important derivatives for Logarithmic Functions:

$$\frac{d}{dx}\left(\log_a x\right) = \frac{1}{x \ln a} \qquad \frac{d}{dx}\left(\ln x\right) = \frac{1}{x}$$

$$\frac{d}{dx}\left(\ln g(x)\right) = \frac{g'(x)}{g(x)} \qquad \frac{d}{dx}\left(\ln|x|\right) = \frac{1}{x}$$

Example 1: Differentiate each function:

a. $\ln\left(x^3+1\right)$ use $\ln g(x)$ rule

$$\frac{3x^2}{x^3+1}$$

b. $f(x) = \sqrt{\ln x}$

$\sqrt{u}$ $\quad u = \ln x$ $\quad \frac{1}{2\sqrt{u}}$ $\quad u' \frac{1}{x}$ $=$ $\boxed{\frac{1}{2x\sqrt{\ln x}}}$

~~$\ln x^{1/2}$~~ $\quad \ln u \quad u = x^{1/2}$ $\quad \frac{d}{u} \quad u' = \frac{1}{2\sqrt{x}}$

c. $f(x) = \log_{10}(2 + \sin x)$

$$\frac{1}{(2+\sin x)\ln 10}(\cos x)$$

$$= \frac{\cos x}{(2+\sin x)\ln 10}$$

d. $y = \ln\dfrac{x+1}{\sqrt{x-2}}$

$$y = \ln(x+1) - \tfrac{1}{2}\ln(x-2)$$

$$y' = \frac{1}{x+1} - \frac{1}{2x-4}$$

$$y' = \frac{2x-4-x-1}{(x+1)(2x-4)} = \boxed{\frac{x-5}{(x+1)(2x+4)}}$$

Logarithmic Differentiation: Taking logs in order to simplify complicated functions.

Example 2: Differentiate $y = x^{\sqrt{x}}$

$$\ln y = \sqrt{x}\ln x$$

$$\frac{y'}{y} = \frac{\ln x}{2\sqrt{x}} + \frac{\sqrt{x}}{x}$$

$$\boxed{y' = \left(\frac{x\ln x + 2x}{2x\sqrt{x}}\right)x^{\sqrt{x}}}$$

Homework: day 1 page 226 (1-13)
day 2 page 226 (14, 16, 21, 25, 33-38, 44)