Sec 3.7 day 2 page 226 (14, 16, 21, 25, 33-38, 44)

(14) $F(y) = y \ln(1+e^y)$

$$F'(y) = \ln(1+e^y) + \frac{ye^y}{1+e^y}$$

(16) $H(z) = \ln\sqrt{\frac{a^2-z^2}{a^2+z^2}} = \frac{1}{2}\ln(a^2-z^2) - \frac{1}{2}\ln(a^2+z^2)$

$$H'(z) = \frac{-z}{a^2-z^2} - \frac{z}{a^2+z^2} = \frac{-a^2z - z^3 - a^2z + z^3}{a^4-z^4}$$

$$= \frac{-2a^2z}{a^4-z^4} \text{ or } \frac{2a^2z}{z^4-a^4}$$

(21) $y = x^2 \ln 2x$

$$y' = 2x\ln 2x + \frac{2x^2}{2x} = 2x\ln 2x + x = y'$$

$$y'' = 2\ln 2x + 2x\left(\frac{2}{2x}\right) + 1$$

$$y'' = 2\ln 2x + 3$$

(25) $y = \ln(x^2 - 3x + 1)$, $(3, 0)$

$$y' = \frac{2x-3}{x^2-3x+1} \qquad y'(3) = \frac{6-3}{9-9+1} = 3$$

$$y = 3(x-3) \Rightarrow y = 3x - 9$$

(33) $y = (2x+1)^5 (x^4-3)^6$

$\ln y = 5\ln(2x+1) + 6\ln(x^4-3)$

$\frac{y'}{y} = \frac{10}{2x+1} + \frac{24x^3}{x^4-3}$

$y' = \frac{10x^4-30+48x^4+24x^3}{(2x+1)(x^4-3)} (2x+1)^5 (x^4-3)^6$

$$\boxed{y' = (58x^4+24x^3-30)(2x+1)^4(x^4-3)^5}$$

(34) $y = \sqrt{x}\, e^{x^2} (x^2+1)^{10}$

$\ln y = \frac{1}{2}\ln x + \ln e^{x^2} + 10\ln(x^2+1)$

$\frac{y'}{y} = \frac{1}{2x} + 2x + \frac{20x}{x^2+1}$

$$y' = \left(\frac{1}{2x} + 2x + \frac{20x}{x^2+1}\right)\left(\sqrt{x}\, e^{x^2} (x^2+1)^{10}\right)$$

(35) $y = \frac{\sin^2 x \tan^4 x}{(x^2+1)^2}$

$\ln y = 2\ln\sin x + 4\ln\tan x - 2\ln(x^2+1)$

$\frac{y'}{y} = \frac{2\cos x}{\sin x} + \frac{4\sec^2 x}{\tan x} - \frac{4x}{x^2+1}$

$$\boxed{y' = \left(2\cot x + \frac{4\sec^2 x}{\tan x} - \frac{4x}{x^2+1}\right)\left(\frac{\sin^2 x \tan^4 x}{(x^2+1)^2}\right)}$$

(36) $y = \sqrt[4]{\frac{x^2+1}{x^2-1}}$

$\ln y = \frac{1}{4}\ln(x^2+1) - \frac{1}{4}\ln(x^2-1)$

$\frac{y'}{y} = \frac{2x}{2^4(x^2+1)} - \frac{2x}{2^4(x^2-1)}$

$y' = \frac{1}{2}\left(\frac{x}{x^2+1} - \frac{x}{x^2-1}\right) = \frac{1}{2}\left(\frac{x^3-x-x^3-x}{(x^4-1)}\right)$

$y' = \frac{1}{2}\left(\frac{-2x}{(x^4-1)}\right)$

$$\boxed{y' = \left(\frac{-x}{(x^4-1)}\right)\sqrt[4]{\frac{x^2+1}{x^2-1}}}$$

(37) $y = x^x$

$\ln y = x\ln x$

$\frac{y'}{y} = \ln x + \frac{x}{x} \Rightarrow \boxed{y' = (\ln x + 1)(x^x)}$

(38) $y = x^{\cos x}$

$\ln y = \cos x \ln x$

$\frac{y'}{y} = -\sin x \ln x + \frac{\cos x}{x}$

$$\boxed{y' = \left(\frac{-x\sin x\ln x + \cos y}{x}\right)x^{\cos x}}$$

(44) Find y' if $x^y = y^x$

$y \ln x = x \ln y$

$y' \ln x + \frac{y}{x} = \ln y + \frac{y'x}{y}$

$xy\,y' \ln x + y^2 = xy \ln y + y'x^2$

$y'(xy \ln x - x^2) = xy \ln y - y^2$

$$y' = \frac{xy \ln y - y^2}{xy \ln x - x^2}$$