Use Ex 1 as pod using the old method

Name Answer Key
Date 2012/2013 Period

Section 3.5 notes

Implicit differentiation: we differentiate both sides of the equation with respect to x and solve for y'.

How do we differentiate y?

$y = 2x+1$ → $y^2 = (2x+1)^2$

$y' = 2$ $\quad (y^2)' = \underbrace{2(2x+1)}_{2y}\underbrace{(2)}_{y'}$

$(y^2)' = 2yy'$ $\quad \therefore$ use chain rule and always add a y' after we differentiate a y.

Example 1: If $x^2 + y^2 = 25$, find $\frac{dy}{dx}$

$\frac{dy}{dx} = 2x + 2yy' = 0$ $\quad \Rightarrow \quad y^2 = 25 - x^2$

$2yy' = -2x$ $\quad y = \sqrt{25-x^2}$

$y' = \frac{-2x}{2y} = -\frac{x}{y}$

old method $\therefore \frac{-2x}{2\sqrt{25-x^2}} = \frac{-x}{\sqrt{25-x^2}}$

$y' = \frac{-x}{\sqrt{25-x^2}}$ ← same →

Example 2: If $x^3 + y^3 = 6xy$, find $\frac{dx}{dy}$

$\frac{dx}{dy} = 3x^2 + 3y^2y' = 6y + 6xy'$

$3y^2y' - 6xy' = 6y - 3x^2$

$y'(3y^2 - 6x) = 6y - 3x^2$

$y' = \frac{6y-3x^2}{3y^2-6x} = \boxed{\frac{2y-x^2}{y^2-2x} = y'}$

Example 3: Find y' if $\sin(x+y) = y^2 \cos x$

$(1+y')(\cos(x+y)) = 2yy'\cos x - y^2 \sin x$

$\cos(x+y) + y'\cos(x+y) = 2yy'\cos x - y^2 \sin x$

$y'\cos(x+y) - 2yy'\cos x = -y^2 \sin x - \cos(x+y)$

$y'(\cos(x+y) - 2y\cos x) = -y^2 \sin x - \cos(x+y)$

$$\boxed{y' = \frac{-y^2 \sin x - \cos(x+y)}{\cos(x+y) - 2y\cos x}}$$

Day 2

Example 4: Find y'' of $x^4 + y^4 = 16$

$y' \Rightarrow \quad 4x^3 + 4y^3 y' = 0$

$y' = -\frac{4x^3}{4y^3} = \frac{-x^3}{y^3}$

$$y'' = \frac{-3x^2 y^3 - 3y^2 y'(-x^3)}{y^6}$$

$$y'' = \frac{-3x^2 y^3 + 3x^3 y^2 y'}{y^6}$$

$$y'' = \frac{-3x^2 y^3 + 3x^3 y^2 \left(\frac{-x^3}{y^3}\right)}{y^6}$$

$$y'' = \frac{-3x^2 y^3 - \frac{3x^6}{y}}{y^6}$$

$$\boxed{y'' = \frac{-3x^2 y^4 - 3x^6}{y^7}} \rightarrow$$

Orthogonal: Two curves are orthogonal if at each point of intersection their tangent lines are perpendicular.

Example 4: Show that $xy = c \quad c \neq 0$ and $x^2 - y^2 = k \quad k \neq 0$ are orthogonal.

$xy = c$

$y + xy' = 0$

$y_1' = \frac{-y}{x}$

$x^2 - y^2 = k$

$2x - 2yy' = 0$

$y_2' = \frac{-2x}{-2y} = \frac{x}{y}$

because $y_1' = \frac{-y}{x}$ and $y_2' = \frac{x}{y}$ their product is equal to -1 $\therefore$ they are perpendicular at every point.

Homework: day 1 page 214 (1, 3-8)
Day 2 page 214 (10-12, 17, 18, 22, 23, 31, 41, 42)

$$= -\frac{3x^2(y^4+x^4)}{y^7} \Rightarrow y^4+x^4=16 \Rightarrow \frac{-3x^2(16)}{y^7}$$

$$\boxed{y'' = -\frac{48x^2}{y^7}}$$