Sec 3.5 page 214 (10-12, 17, 18, 22, 23, 31, 41, 42)

(10) $1 + x = \sin(xy^2)$

$1 = \cos(xy^2)(y^2 + 2xyy')$

$1 = y^2\cos(xy^2) + 2xyy'\cos(xy^2)$

$1 - y^2\cos(xy^2) = 2xyy'\cos(xy^2)$

$\boxed{\dfrac{1 - y^2\cos(xy^2)}{2xy\cos(xy^2)} = y'}$

(11) $4\cos x \sin y = 1$

$-4\sin x \sin y + 4y'\cos x \cos y = 0$

$y' = \dfrac{4\sin x \sin y}{4\cos x \cos y} = \boxed{\tan x \tan y}$

(12) $y\sin(x^2) = x\sin(y^2)$

$y'\sin(x^2) + y2x\cos(x^2) = \sin(y^2) + 2yy'x\cos(y^2)$

$y'(\sin(x^2) - 2yx\cos(y^2)) = \sin(y^2) - y2x\cos(x^2)$

$\boxed{y' = \dfrac{\sin(y^2) - y2x\cos(x^2)}{\sin(x^2) - 2yx\cos(y^2)}}$

(17) $f(x) + x^2[f(x)]^3 = 10 \qquad f(1) = 2 \quad$ Find $f'(1)$

$f'(1) + 2x[f(1)]^3 + x^2(3f(1)^2)f'(1) = 0$

$= f'(1) + 2(1)(2)^3 + 1^2(3(2)^2)f'(1) = 0$

$= f'(1)(1 + 12) = -16$

$f'(1) = \boxed{\dfrac{-16}{13}}$

18. If $g(x) + x\sin g(x) = x^2$, find $g'(0)$

$$g'(x) + \sin g(x) + x g'(x)\cos g(x) = 2x$$

$$g'(x)(1 + x\cos g(x)) = 2x - \sin g(x)$$

$$g'(x) = \frac{2x - \sin g(x)}{1 + x\cos g(x)} \qquad x = 0$$

$$g'(0) = \frac{2(0) - \sin g(0)}{1 + 0\cos g(0)} = -\sin g(0)$$

or b/c $g(0) + 0\sin g(0) = 0^2$

$g(0) = 0$ so $-\sin g(0) = -\sin 0 = 0$

22. $\sin(x+y) = 2x - 2y \qquad (\pi, \pi)$

$$(1+y')\cos(x+y) = 2 - 2y'$$

$$\cos(x+y) + y'\cos(x+y) = 2 - 2y'$$

$$y'(\cos(x+y) + 2) = 2 - \cos(x+y)$$

$$y' = \frac{2 - \cos(x+y)}{\cos(x+y) + 2} \qquad y'(\pi,\pi) = \frac{2 - \cos(2\pi)}{\cos(2\pi) + 2} = \frac{1}{3}$$

$$\boxed{\therefore\ y - \pi = \frac{1}{3}(x - \pi)}$$

(23) $x^2 + xy + y^2 = 3 \quad (1,1)$

$2x + y + xy' + 2yy' = 0$

$y' = \dfrac{-2x - y}{x + 2y} \qquad y'(1,1) = \dfrac{-2(1) - 1}{1 + 2(1)} = \dfrac{-3}{3} = -1$

$\boxed{\therefore\ y - 1 = -1(x - 1) \text{ or } y = -x + 2}$

(31) $9x^2 + y^2 = 9$

$18x + 2yy' = 0$

$y' = \dfrac{-18x}{2y} = \dfrac{-9x}{y}$

$y'' = \dfrac{-9y + 9xy'}{y^2} \Rightarrow y'' = \dfrac{-9y + 9x\left(\frac{-9x}{y}\right)}{y^2}$

$y'' = \dfrac{-9y^2 - 81x^2}{y^3} = \dfrac{-9\overbrace{(y^2 + 9x^2)}^{=9}}{y^3} = \boxed{\dfrac{-81}{y^3}}$

(41) $x^2 + y^2 = r^2 \qquad ax + by = 0 \qquad$ solve equation 2 for a

$2x + 2yy' = 0 \qquad a + by' = 0 \qquad a = \dfrac{-by}{x}$ plug in

$y' = \dfrac{-x}{y} \qquad y' = -\dfrac{a}{b}$

$y' = \dfrac{-1}{b}\left(\dfrac{-by}{x}\right) = \dfrac{y}{x}$

Opposite reciprocals

42) $x^2+y^2=ax$

$2x+2yy'=a$

$y'=\dfrac{a-2x}{2y}$

Solve for a

$a=\dfrac{x^2+y^2}{x}$

plug in to y'

$y'=\dfrac{\frac{x^2+y^2}{x}-2x}{2y}$

$y'=\dfrac{x^2+y^2-2x^2}{2xy}$

$y'=\dfrac{-x^2+y^2}{2xy}$

$x^2+y^2=by$

$2x+2yy'=by'$

$y'=\dfrac{-2x}{b-2y}$

solve for b

$b=\dfrac{x^2+y^2}{y}$

plug in to y'

$y'=\dfrac{-2x}{\frac{x^2+y^2}{y}-2y}$

$y'=\dfrac{-2x}{\frac{x^2+y^2-2y^2}{y}}$

$y'=\dfrac{-2xy}{-x^2+y^2}$

Opposite reciprocals