

Name \_\_\_\_\_

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 Sec 3.7 Calculus: Chain Rule

Function	Outer Function (f(u))	Inner Function (g(x))	Derivative using Chain rule
$F(x) = \sqrt{x^3 - 1}$	$\sqrt{u} = u^{1/2}$	$x^3 - 1$	
Individual Derivative	$\frac{1/2 u^{-1/2}}{\frac{1}{2\sqrt{x^3-1}}}$	$3x^2$	$\frac{3x^2}{2\sqrt{x^3-1}}$
$F(x) = \frac{1}{\sqrt{2x^2+2}}$	$u^{-1/2}$	$2x^2 + 2$	
Individual Derivative	$\frac{-1/2 u^{-3/2}}{\frac{-1}{2\sqrt{(2x^2+2)^3}}}$	$4x$	$\frac{-2x}{\sqrt{(2x^2+2)^3}}$
$F(x) = \cos(x^2)$	$\cos u$	$x^2$	
Individual Derivative	$\frac{-\sin u}{-\sin(x^2)}$	$2x$	$-2x \sin(x^2)$
$F(x) = \cos^2 x$	$u^2$	$\cos x$	
Individual Derivative	$\frac{2u}{2 \cos x}$	$-\sin x$	$-2 \cos x \sin x$ $= -\sin 2x$

1. Determine the outer function and write in terms of  $u$ .
2. Determine the inner function.
3. Find the derivative of the outer function in terms of  $u$ .
4. Replace the  $u$  from number 3 with the inner function.
5. Find the derivative of the inner function.
6. Multiply 4 and 5 together.

Example: Find

$F'(x)$ if $F(x) = \sqrt{x^2 + 1}$ .	$F'(x) = \sin(x^2)$	$F'(x) = \sin^2 x$
Step 1: $\sqrt{u}$	$\sin u$	$u^2$
Step 2: $x^2 + 1$	$x^2$	$\sin x$
Step 3: $\frac{1}{2}u^{-1/2}$	$\cos u$	$2u$
Step 4: $\frac{1}{2\sqrt{x^2+1}}$	$\cos(x^2)$	$2 \sin x$
Step 5: $2x$	$2x$	$\cos x$
Step 6: $\frac{x}{\sqrt{x^2+1}}$	$2x \cos(x^2)$	$2 \sin x \cos x$
$\therefore F'(x) =$		$= \sin 2x$

Section 3.~~5~~<sup>4</sup> notes

Day one

Chain

**Power Rule:** for any composition function is

$$(f \circ g)'(x) = f'(g(x))g'(x)$$

In Leibniz notation: if  $y = f(u)$  and  $u = g(x)$  then

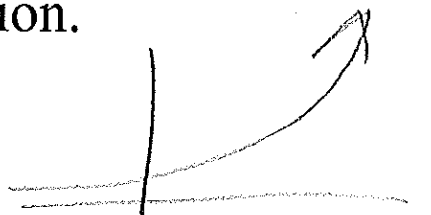
$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Day two

Differentiating the power function.

$$\frac{d}{dx}(a^x) = a^x \ln a$$

$$e^x \ln e = e^x$$



Example: Differentiate  $2^x$ .

$$\frac{d}{dx} = 2^x \ln 2$$

### Example 2

$$g(x) = (2+3x)^2 (2+x-2x^2)^5$$

$$2(2+3x)(3x)(2+x-2x^2)^5 + 5(2+x-2x^2)^4 (1-4x)(2+3x)$$

$$(2+3x)(2+x-2x^2)^4 (6x(2+x-2x^2) + (5-20x)(2+3x))$$

$$12x + 6x^2 - 12x^3 + 10 - 40x + 15x^2 - 40x^3$$

$$(2+3x)(2+x-2x^2)^4 (-12x^3 - 54x^2 - 13x + 10)$$

### Example 3

$$y = \tan^2(3\theta)$$

$$y = u^2 \quad u = \tan(3\theta)$$

$$y' = 2u \quad u' = 3 \sec^2 3\theta$$

$$2(\tan(3\theta))$$

$$6 \tan(3\theta) (\sec^2(3\theta))$$