Sec 3.4 page 205 (1-6) day1

① $y = \sqrt[3]{1+4x}$

$f = u^{1/3}$ $\quad u = 1+4x$

$f' = \frac{1}{3}u^{-2/3}$ $\quad u' = 4$

$y' = \frac{1}{3\sqrt[3]{(1+4x)^2}} \cdot 4 = \boxed{\frac{4}{3\sqrt[3]{(1+4x)^2}}}$

② $y = (2x^3+5)^4$

$f = u^4$ $\quad u = 2x^3+5$

$f' = 4u^3$ $\quad u' = 6x^2$

$y' = 4(2x^3+5)^3(6x^2)$

$= \boxed{24x^2(2x^3+5)^3}$

③ $y = \tan \pi x$

$f = \tan u$ $\quad u = \pi x$

$f' = \sec^2 u$ $\quad u' = \pi$

$y' = \sec^2(\pi x)(\pi)$

$= \boxed{\pi \sec^2(\pi x)}$

④ $y = \sin(\cot x)$

$f = \sin u$ $\quad u = \cot x$

$f' = \cos u$ $\quad u' = -\csc^2 x$

$y' = \cos(\cot x)(-\csc^2 x)$

$= \boxed{-\csc^2 x \cos(\cot x)}$

⑤ $y = e^{\sqrt{x}}$

$f = e^u$ $\quad u = \sqrt{x}$

$f' = e^u$ $\quad u' = \frac{1}{2\sqrt{x}}$

$y' = e^{\sqrt{x}}\left(\frac{1}{2\sqrt{x}}\right) = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

⑥ $y = \sqrt{2 - e^x}$

$f = \sqrt{u}$ $\quad u = 2 - e^x$

$f' = \frac{1}{2\sqrt{u}}$ $\quad u' = -e^x$

$$y' = \frac{-e^x}{2\sqrt{2-e^x}}$$

Sec 3.4 day 2 page 205 (7-31 odd)

⑦ $F(x) = (x^4 + 3x^2 - 2)^5$

$f = u^5$ $\quad u = x^4 + 3x^2 - 2$

$f' = 5u^4$ $\quad u' = 4x^3 + 6x$

$$F'(x) = 5(x^4 + 3x^2 - 2)^4(4x^3 + 6x)$$

or

$$F'(x) = (20x^3 + 30x)(x^4 + 3x^2 - 2)^4$$

⑨ $F(x) = \sqrt{1 - 2x}$

$f = \sqrt{u}$ $\quad u = 1 - 2x$

$f' = \frac{1}{2\sqrt{u}}$ $\quad u' = -2$

$$F'(x) = \frac{-2}{2\sqrt{1-2x}} = \frac{-1}{\sqrt{1-2x}}$$

⑪ $f(z) = \frac{1}{z^2 + 1}$

$f = u^{-1}$ $\quad u = z^2 + 1$

$f' = -u^{-2}$ $\quad u' = 2z$

$$f'(z) = -(z^2+1)^{-2}(2z) = \frac{-2z}{(z^2+1)^2}$$

⑬ $y = \cos(a^3 + x^3)$

$f = \cos u$ $\quad u = a^3 + x^3$

$f' = -\sin u$ $\quad u' = 3x^2$

$$y' = -\sin(a^3 + x^3)(3x^2) = -3x^2 \sin(a^3 + x^3)$$

(15) $h(t) = t^3 - 3^t$

$\boxed{h'(t) = 3t^2 - 3^t \ln 3}$

(17) $y = x e^{-kx}$ product rule and chain rule

product

$f = x \quad g = e^{-kx} \Rightarrow$ chain rule $f = e^u \quad u = -kx$

$f' = 1 \quad g' = e^{-kx}(-k) \qquad f' = e^u \quad u' = -k$

$y' = e^{-kx} - kx e^{-kx} = \boxed{e^{-kx}(1 - kx)}$

(19) $y = (2x-5)^4 (8x^2-5)^{-3}$ product and chain rule

product

$f = (2x-5)^4 \quad g = (8x^2-5)^{-3}$

chain $f' = 8(2x-5)^3$ chain $g' = -48(8x^2-5)^{-4}$

$$\boxed{y' = \frac{8(2x-5)^3}{(8x^2-5)^3} - \frac{48(2x-5)^4}{(8x^2-5)^4}}$$

(21) $y = e^{x \cos x}$ chain and product

chain rule

$f = e^u \quad u = x\cos x$

$f' = e^u \quad u' = \cos x - x \sin x$

↑ product rule

$$\boxed{y' = e^{x\cos x}(\cos x - x \sin x)}$$

(23) $y = \left(\frac{x^2+1}{x^2-1}\right)^3$ chain and quotient rule

Chain rule

$s = u^3$ $\quad u = \frac{x^2+1}{x^2-1}$

$s' = 3u^2$

$u' = \frac{2x(x^2-1) - 2x(x^2+1)}{(x^2-1)^2} = \frac{2x^3 - 2x - 2x^3 - 2x}{(x^2-1)^2}$ (quotient rule)

$= \frac{-4x}{(x^2-1)^2}$

$y' = 3\left(\frac{x^2+1}{x^2-1}\right)^2 \left(\frac{-4x}{(x^2-1)^2}\right) = \boxed{\frac{-12x(x^2+1)^2}{(x^2-1)^4}}$

(25) $y = \sec^2 x + \tan^2 x$ chain rule for both terms

$s = u^2$ $\quad u = \sec x$ $\quad s = u^2$ $\quad u = \tan x$

$s' = 2u$ $\quad u' = \sec x \tan x$ $\quad s' = 2u$ $\quad u' = \sec^2 x$

$y' = 2\sec x(\sec x \tan x) + 2\tan x(\sec^2 x)$

$= \boxed{4\sec^2 x \tan x}$

(27) $y = \frac{r}{\sqrt{r^2+1}}$ quotient and chain rule

$s = r$ $\quad g = \sqrt{r^2+1}$

$s' = 1$ $\quad g' = \frac{2r}{2\sqrt{r^2+1}} = \frac{r}{\sqrt{r^2+1}}$

$y' = \frac{\sqrt{r^2+1} - \frac{r^2}{\sqrt{r^2+1}}}{r^2+1} = \frac{r^2+1-r^2}{(r^2+1)(\sqrt{r^2+1})} = \boxed{\frac{1}{(r^2+1)(\sqrt{r^2+1})}}$

(29) $y = \sin(\tan 2x)$ chain within chain

$S = \sin u$ $\quad u = \tan 2x$

$S' = \cos u$ $\quad u' = 2\sec^2 2x$

$y' = \cos(\tan 2x)(2\sec^2(2x)) = \boxed{2\cos(\tan(2x))(\sec^2(2x))}$

(31) $y = 2^{\sin \pi x}$

$S = 2^u$ $\quad u = \sin \pi x$

$S' = 2^u \ln 2$ $\quad u' = \pi \cos \pi x$

$\boxed{y' = 2^{\sin \pi x} \ln 2 (\pi \cos \pi x)}$