Name Answer Key
Period ______ Date 2012/2013

Calculus Section 3.1-3.4 Review

Differentiate.

1. $y = 3x^4 - 5x^2 + 2x - 181$

$12x^3 - 10x + 2$

2. $y = \sqrt{x} + \dfrac{1}{\sqrt[3]{x^4}}$

$x^{1/2} + x^{-4/3} = \frac{1}{2}x^{-1/2} - \frac{4}{3}x^{-7/3}$

$$\boxed{\frac{1}{2\sqrt{x}} - \frac{4}{3x^2\sqrt[3]{x}}}$$

$\frac{1}{2\sqrt{x}} - \frac{4}{3}(x^{-7/3})$

3. $y = \dfrac{3x-2}{1-x^2}$

$f = 3x-2 \quad g = 1-x^2$

$f' = 3 \quad g' = -2x$

$$\frac{3-3x^2+(+2x(3x-2))}{(1-x^2)^2}$$

$$\boxed{= \frac{3x^2-4x+3}{(1-x^2)^2}}$$

4. $y = \dfrac{\sin x}{x^4}$

$f = \sin x \quad g = x^4$

$f' = \cos x \quad g' = 4x^3$

$$\frac{x^3(x\cos x - 4\sin x)}{x^8}$$

$$\boxed{= \frac{(x\cos x - 4\sin x)}{x^5}}$$

5. Given f(5) = 2, f'(5) = 2,1, g(5) = -3, and g'(5) = 6.12, find the value of

a. (5f – g)'(5) b. (fg)'(5) c. (f/g)'(5)

a. $5f' - g'$

$= 5(2.1) - 6.12 = 4.38$

b. $f'g + g'f$

$2.1(-3) + 6.12(2)$

$= 5.94$

c. $\frac{f'g - g'f}{g^2} = \frac{2.1(-3) - 6.12(2)}{(-3)^2}$

$= -2.06$

6. Find and write the equation of the tangent line and the normal line to the curve $y = 6x^2 - 2x + 1$ at the point (2, 21).

$m = f' = 12x - 2$

$f'(2) = 12(2) - 2 = 22$

tangent =

$y - 21 = 22(x-2)$

or $y = 22x - 23$

normal =

$y - 21 = \frac{-1}{22}(x-2)$

or $y = \frac{-x}{22} + 21\frac{1}{11}$

7. The position function for a particle is $s(t) = -12t^2 + 36t$, where s is measure in feet and t is measure in seconds:

a. Find the velocity when t = 1, t = 2, and t = 3.

$s'(t) = v(t) = -24t + 36$

$v(1) = 12$ ft/sec

$v(2) = -12$ ft/sec

$v(3) = -36$ ft/sec

b. When does the velocity equal zero?

$v(t) = 0 \qquad -24t + 36 = 0$

$t = 36/24 = 3/2$ so at 1.5 sec

c. When does the velocity equal 12 ft/s?

$v(1) = 12$ ft/s $\qquad -24t + 36 = 12$

so at 1 sec.

Find the derivative of each function. Be sure to state your inner and outer function.

8. $y = (1 - 9x^2)^{12}$

$u^{12} \qquad u = 1 - 9x^2$

$12u^{11} \qquad u' = -18x$

$12(1-9x^2)^{11}$

$-18x(12)(1-9x^2)^{11}$

$\boxed{= -216x(1-9x^2)^{11}}$

9. $y = e^{-5x} \sin 3x$

$f = e^{-5x} \qquad g = \sin 3x$

$f' = -5e^{-5x} \qquad g' = 3\cos 3x$

$-5e^{-5x} \sin 3x + 3e^{-5x} \cos 3x$

$\boxed{= e^{-5x}(3\cos 3x - 5\sin 3x)}$

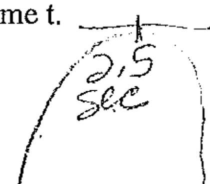
10. The population of a bacteria colony after t hours is given by the function $p(t) = 25 + 2t + 3t^2$. Find the growth rate at 4 hours.

$p'(t) = 2 + 6t$

$p'(4) = 2 + 24 = \boxed{26 \text{ bac/hr}}$

11. Suppose that a ball is thrown straight upward and that its height (in feet) is given by the formula $h(t) = 70t - 14t^2$ (where t is time in seconds).

a. Find the instantaneous velocity and acceleration of the ball at time t.



$v(t) = 70 - 28t$

$a(t) = -28$

b. What is the maximum height obtained by the ball?

$v(t) = 0$

$70 - 28t = 0$

$70 = 28t$

$\boxed{t = 2.5 \text{ sec}} \Rightarrow \boxed{h(2.5) = 87.5 \text{ ft}}$

c. What is the average velocity of the ball during the time interval t = 2 to t = 5?

$\dfrac{h(5) - h(2)}{5 - 2} = \dfrac{0 - 84}{3} = \boxed{-28 \text{ ft/s}}$

d. How long does it take before the ball lands?

5 sec

e. At what time is the height of the ball 42 feet? 56 feet?

$70t - 14t^2 = 42$

$70t - 14t^2 - 42 = 0$

$\boxed{t \approx .697 \quad t \approx 4.303}$

$70t - 14t^2 = 56$

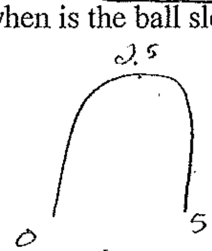
$70t - 14t^2 - 56 = 0$

$\boxed{t = 1, \ t = 4}$

f. When is the ball speeding up and when is the ball slowing down?

SU (2.5, 5)

SD (0, 2.5)



12. If $f(x) = 10^{9-4x^2}$, find $f'(1)$

$10^u \quad u = 9 - 4x^2$

$10^u \ln 10 \quad u' = -8x$

$10^{9-4x^2} \ln 10$

$-8x(10^{9-4x^2}) \ln 10$

$f'(1) = -8(10^5) \ln 10$

$\boxed{= -800000 \ln 10}$

13. Find an equation of the tangent to $y = (2+7x)^4$ at the point (0, 16).

$u^4 \quad u = 2 + 7x$

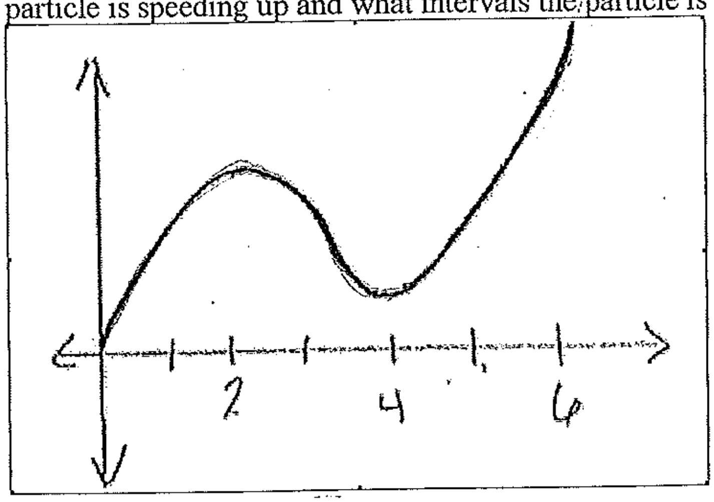
$4u^3 \quad u' = 7$

$f' = 28(2+7x)^3$

$f'(0) = 28(2)^3 = 224$

$\boxed{y = 224x + 16}$

14. The graph of the position function of a particle is given. Tell what intervals the particle is speeding up and what intervals the particle is slowing down.



SU (2,3) (4,6)

SD (0,2) (3,4)