Name *Answer Key*

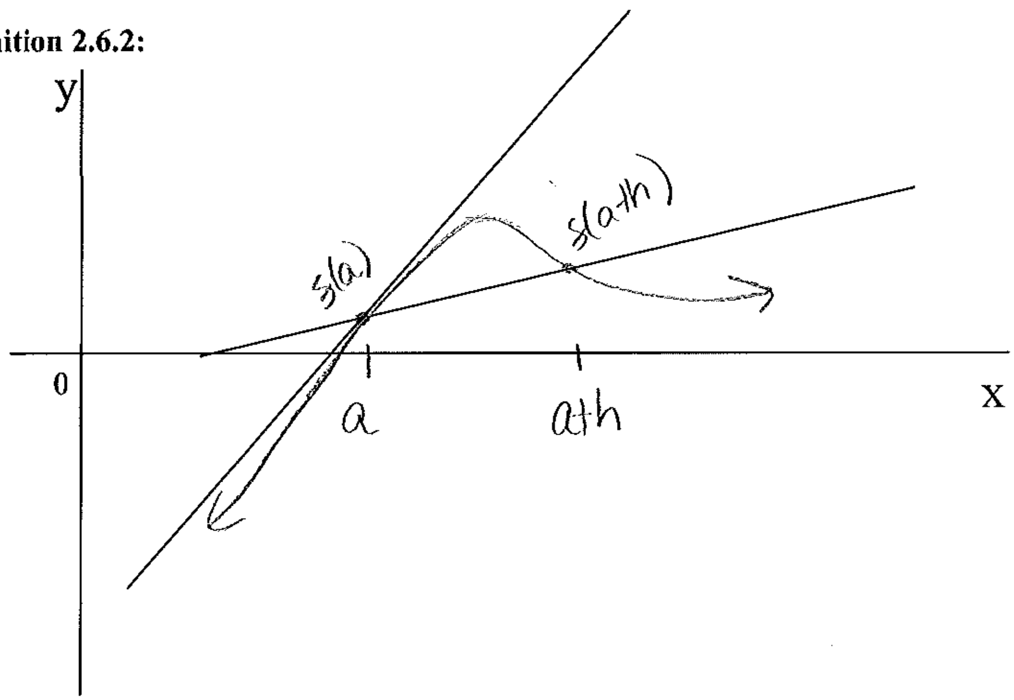
Sec 2.6 Notes

Definition 2.6.1: the tangent line to the curve $y = f(x)$ at the point $P(a, f(a))$ is the line through P with slope

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

provided the limit exists.

Definition 2.6.2:



$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Definition 2.6.3: The **derivative** of a function f at a number a is found by

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

if the limit exists.

Definition 2.6.4: another way to say def from above:

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

↓ fix

Example 1: Find the equation of the line tangent to the parabola $y = x^2$ at the point $P(1, 1)$.

$$m = \lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \frac{(x-1)(x+1)}{(x-1)} = 2$$

$$y - 1 = 2(x-1) \Rightarrow \boxed{y = 2x - 1}$$

Example 2: Find an equation of the tangent line to the hyperbola $y = \frac{3}{x}$ at the point (3, 1).

$$\lim_{h \to 0} \frac{\frac{3}{x+h} - \frac{3}{x}}{h} = \frac{3x - 3x - 3h}{hx(x+h)} = \frac{-3}{x^2+h} = -\frac{3}{x^2}$$

$$x = 3 = \frac{-3}{x^2} = \frac{-3}{9} = \frac{-1}{3} \quad y - 1 = -\frac{1}{3}(x-3)$$

$$\Rightarrow \boxed{y = \frac{-1}{3}x + 2}$$

Example 3:

a. Find the derivative of the function $f(x) = x^2 - 8x + 9$.

b. Find an equation of the tangent line to the parabola $f(x)$ at the point (3, -6).

$$\lim_{h \to 0} \frac{(x+h)^2 - 8(x+h) + 9 - x^2 + 8x - 9}{h} =$$

$$\lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{-8x} - 8h \cancel{+9}}{h} = \lim_{h \to 0} 2x + h - 8 = \boxed{2x - 8}$$

$$x = 3 \Rightarrow 2(3) - 8 = -2$$

$$y + 6 = -2(x-3)$$

$$\boxed{y = -2x}$$

Homework day 1: page 145 (1, 3, 5, 6, 8, 9, 11, 12, 23, 24, 26)

Sec 2.6 day 2

We found that average velocity is equivalent to the equation $\dfrac{f(a+h)-f(a)}{h}$.

Definition 2.6.5: Instantaneous velocity can be found by

$$v(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

Fix

Example 1: Suppose that a ball is dropped from the upper observation deck of the CN Tower in Toronto, 450 m above the ground. $s(t)=4.9t^2$ Equation of motion

a. What is the velocity of the ball after 5 seconds?

$\lim_{h\to 0}\dfrac{4.9(5+h)^2-122.5}{h}=\boxed{49\,m/s}$

b. How fast is the ball traveling when it hits the ground?

$450=4.9t^2$

$t\approx 9.6\text{ sec}$

$\lim_{h\to 0}\dfrac{4.9(9.6+h^2)-451.584}{h}$

$94.08+4.9h=\boxed{94.08\,m/s}$

Instantaneous rate of change: is the derivative of a function.

Speed: of a particle is the absolute value of the velocity, $|f'(a)|$.

Example 2: The position of a particle is given by the equation of motion $s=f(t)=\dfrac{1}{1+t}$, where t is measured in seconds and s in meters. Find the velocity and speed after 2 seconds.

$f'(2)=\lim_{h\to 0}\dfrac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\dfrac{\frac{1}{1+2+h}-\frac{1}{3}}{h}=\lim_{h\to 0}\dfrac{\frac{1}{3+h}-\frac{1}{3}}{h}$

$\lim_{h\to 0}\dfrac{3-3-h}{h(9+3h)}=\boxed{\dfrac{-1}{9}\,m/s}$ $\boxed{\text{Speed}=\frac{1}{9}\,m/s}$

Example 3: The cost of producing x ounces of gold from a new gold mine is $C=f(x)$ dollars.

a. What is the meaning of $f'(x)$? What are its units? rate of change at which the cost is changing per ounce of gold produced. Units \$/oz

b. What does the statement $f'(800)=17$ mean? that after producing 800 oz of gold, the cost per ounce is \$17. ⇒ \$17/oz

c. What will happen to the values of $f'(x)$ over the short term? Long term?
Short term $f(x)$ will decrease, eventually it may increase as to extract the gold will become increasingly difficult with time

Definition 2.6.4: instantaneous rate of change is equal to

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Example 4: The displacement (in meters) of a particle moving in a straight is given by $s = t^2 - 8t + 18$, where t is measured in seconds.

a. Find the average velocity over each time interval.

i. [3, 4] ii. [4, 5]

i) $\frac{S(4) - S(3)}{4-3} = \boxed{-1\, m/s}$ $\qquad \frac{S(5) - S(4)}{5-4} = \boxed{1\, m/s}$

b. Find the instantaneous velocity when $t = 4$.

$\lim_{h \to 0} \frac{(4+h)^2 - 8(4+h) + 18 - 2}{h} = \lim_{x \to 0} \frac{\cancel{16} + 8h + h^2 - \cancel{32} - 8h + \cancel{16}}{h}$

$\lim_{h \to 0} \frac{-h}{1} = \boxed{0\, m/s}$

What does that mean?

Homework day 2: page 143(13-15, 17-19, 27, 33-36, 42, 43, 47, 48)