Name *Answer Key*
Date *2/12/2013* Period ____

Section 2.5

Infinity notation: $\lim_{x \to a} f(x) = \infty$

This means that $f(x)$ can be made large as x gets very close to a but cannot equal a.

Vertical asymptote: written as $x = a$ can exist if at least one of the following are true:

$$\lim_{x \to a} f(x) = \pm\infty \qquad \lim_{x \to a^-} f(x) = \pm\infty \qquad \lim_{x \to a^+} f(x) = \pm\infty$$

Natural Log asymptote: at x = 0 because
$\lim_{x \to 0^+} \ln x = -\infty$

Example 1: Find $\lim_{x \to 3^+} \frac{2x}{x-3}$ and $\lim_{x \to 3^-} \frac{2x}{x-3}$

what happens when you divide a constant by numbers that keep getting closer to zero?

$\lim_{x \to 3^+} \frac{2x}{x-3}$

$\frac{+}{0} \to \infty$

$\lim_{x \to 3^-} \frac{2x}{x-3}$

$\frac{+}{-} \to -\infty$

$\frac{2}{2} = 1$

$\frac{2}{1} = 2$

$\frac{2}{1/2} = 4$

$\frac{2}{1/4} = 8$

$2/\frac{1}{8} = 16$

...

$\to \infty$

Definition 4: f is a function on the interval (a, ∞), then $\lim_{x \to \infty} f(x) = L$. This means $f(x)$ can be made close to L by taking large values of x.

Horizontal asymptotes: The line y = L is called a horizontal asymptote of the curve y = $f(x)$ if either

$\lim_{x \to \infty} f(x) = L$ or $\lim_{x \to -\infty} f(x) = L$

What happens when you divide a constant by a very large number?

Definition 7: If n is a positive integer, then

$$\lim_{x \to \infty} \frac{1}{x^n} = 0 \qquad \lim_{x \to -\infty} \frac{1}{x^n} = 0$$

$\frac{2}{2} = 1$

$\frac{2}{3} = 2/3$

$\frac{2}{4} = 1/2$

$\frac{2}{1000} = .002$

...

goes to 0

Definition 8: $\lim_{x \to -\infty} e^x = 0$

Example 2: Evaluate $\lim\limits_{x\to\infty} \dfrac{3x^2 - x - 2}{5x^2 + 4x + 1}$. $\Rightarrow \lim\limits_{x\to\infty} \dfrac{\underbrace{\frac{3x^2}{x^2}}_{\to 3} - \underbrace{\frac{x}{x^2}}_{\to 0} - \underbrace{\frac{2}{x^2}}_{\to 0}}{\underbrace{\frac{5x^2}{x^2}}_{\to 5} - \underbrace{\frac{4x}{x^2}}_{\to 0} + \underbrace{\frac{1}{x^2}}_{\to 0}}$

$\therefore$ limit would be $3/5$

Example 3: Compute $\lim\limits_{x\to\infty}\left(\sqrt{x^2+1} - x\right)\left(\dfrac{\sqrt{x^2+1} + x}{\sqrt{x^2+1} + x}\right)$

$$\lim_{x\to\infty} \frac{\cancel{x^2}+1 - \cancel{x^2}}{\sqrt{x^2+1} + x} = \lim_{x\to\infty} \frac{1}{\sqrt{x^2+1} + x}$$

$$= \lim_{x\to\infty} \frac{\frac{1}{x}}{\sqrt{\frac{x^2}{x^2} + \frac{1}{x^2}} + \frac{x}{x}} = \frac{0}{\sqrt{1+0} + 1} = \frac{0}{2} = 0$$

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