Name Answer Key
Date 2012/2013 Period ____

Section 2.4 Day 1

Continuity: A function f is continuous at a number a if

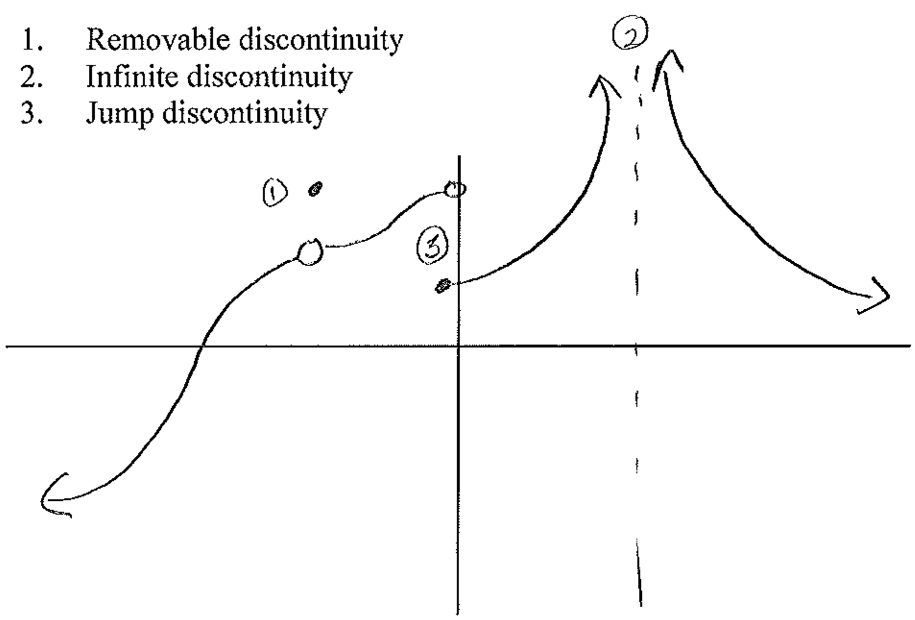
$$\lim_{x \to a} f(x) = f(a).$$

This implies:

1. $f(a)$ is defined. removable
2. $\lim_{x \to a} f(x)$ exists. infinite
3. $\lim_{x \to a} f(x) = f(a)$ jump

Discontinuity: If a graph is discontinuous at a point it can be considered to have one of three things:

1. Removable discontinuity
2. Infinite discontinuity
3. Jump discontinuity



Left and right continuity: A function f is continuous from the right at a number a if

$$\lim_{x \to a^+} f(x) = f(a)$$

and continuous from the left at a number a if

$$\lim_{x \to a^-} f(x) = f(a).$$

Continuous on an interval: A function f is continuous on an interval if it is continuous at every number in the interval.

Example 1:
Show that the function $f(x)=1-\sqrt{1-x^2}$ is continuous on the interval [-1, 1].

① Domain $1-x^2>0$ $1>|x|$ $\therefore D\ [-1,1]$

② $\lim\limits_{x\to a} f(x)$ exist? $\Rightarrow \lim\limits_{x\to -1^+} f(x)=1 \Rightarrow \lim\limits_{x\to 1^-} f(x)=1$

$\therefore \lim\limits_{x\to a} f(x)$ exists

③ $\lim\limits_{x\to a} f(x)=f(a) \Rightarrow \lim\limits_{x\to a} 1-\lim\limits_{x\to a}\sqrt{1-x^2}=\lim\limits_{x\to a} 1-\sqrt{\lim\limits_{x\to a} 1-\lim\limits_{x\to a} x^2}$

$=1-\sqrt{1-a^2}=f(a)$. $\therefore \lim\limits_{x\to a} f(x)=f(a)$, $\therefore f(x)$ is continuous on $[-1,1]$

Theorem 4: If f and g are continuous at a and c is a constant, then the following functions are all continuous at a:

1. $f+g$ 2. $f-g$ 3. cf

4. fg 5. f/g if $g(a)\neq 0$

Theorem 5:
a. any polynomial is continuous everywhere.
b. any rational function is continuous wherever it is defined.

Example 2: Find $\lim\limits_{x\to -2}\dfrac{x^3+2x^2-1}{5-3x}$.

By theorem 5b we know this is a rational function. Therefore it is continuous everywhere it is defined.

D: $5-3x\neq 0 \Rightarrow x\neq 5/3$.

$$\lim_{x\to -2}\frac{x^3+2x^2-1}{5-3x}=\frac{(-2)^3+2(-2)^2-1}{5-3(-2)}=\frac{-1}{11}$$

Homework: p 121 #1-13, 15, 16.

Section 2.4 day 2

Theorem 7: Polynomials, rational, root, trig, inverse trig, exponential, and log functions are all continuous everywhere in their domain:

Theorem 8: $\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right)$

Theorem 9: If g is continuous at a and f is continuous at $g(a)$, then the composite function $f \circ g$ is continuous at a.

Example 3: Where are the following functions continuous?

a. $h(x) = \sin(x^2)$

$h(x) = f(g(x))$

$f(x) = \sin x \quad g(x) = x^2$

$f(x)$ Domain $(-\infty, \infty)$

$g(x)$ Domain $(-\infty, \infty)$

$\therefore$ continuous on $\mathbb{R}$

b. $f(x) = \ln(1 + \cos x)$

because ln can only be taken for + numbers 1 + cosx has to be +

$h(x) = f(g(x))$

$f(x) = \ln x \quad g(x) = 1 + \cos x$

$f(x)$ Domain $(0, \infty) \Rightarrow 1 + \cos x > 0$

$\cos x > -1$

$g(x)$ Domain $(-\infty, \infty)$

$\cos x = -1$ at $\pi, 3\pi, 5\pi$

$\therefore f(x)$ is continuous everywhere except $\pm\pi, \pm 3\pi, \pm 5\pi, \ldots$

Intermediate Value Theorem: (IVT) Suppose f is continuous on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number c in (a, b) such that $f(c) = N$.

Example 4: Show that there is a root of the equation $4x^3 - 6x^2 + 3x - 2 = 0$ between 1 and 2.

$f(1) = 4(1)^3 - 6(1)^2 + 3(1) - 2 = -1$

$f(2) = 4(2)^3 - 6(2)^2 + 3(2) - 2 = 12$

$f(1) < 0 < f(2)$

$\therefore$ by IVT $\exists \{x \mid f(x) = 0\}$

Homework: p 122 #17,19, 20, 22, 28, 33, 35, 41, 42, 51