Name Answer Key
Date 2012/2013 Period J

Section 2.3 Calculating limits using limit laws

Example 1: Evaluate the following limits and justify each step.

a. $\lim_{x\to 5}(2x^2-3x+4)$ $\lim_{x\to 5} 2x^2 - \lim_{x\to 5} 3x + \lim_{x\to 5} 4$

$= 2\lim_{x\to 5} x^2 - 3\lim_{x\to 5} x + 4$

$= 2\left(\lim_{x\to 5} x\right)^2 - 3(5) + 4 \quad = 2(5^2) - 15 + 4$

$= 50 - 15 + 4 = 39$

b. $\lim_{x\to -2}\left(\frac{x^3+2x^2-1}{5-3x}\right)$

$$\frac{\left(\lim_{x\to -2} x\right)^3 + 2\left(\lim_{x\to -2} x\right)^2 - \lim_{x\to -2} 1}{\lim_{x\to -2} 5 - 3\lim_{x\to -2} x}$$

$$= \frac{-8+2(4)-1}{5-3(-2)} = \frac{-8+8-1}{5+6} = -\frac{1}{11}$$

Direct substitution property: If f is a polynomial or a rational function and a is in the domain of f, then

$$\lim_{x\to a} f(x) = f(a).$$

Example 2: Find $\lim_{x\to 1}\frac{x^2-1}{x-1}$. $\lim \frac{1^2-1}{1-1} = \frac{0}{0}$ ← can't have

Factor $\frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}} = x+1 = 2$

Homework: page 111 #1, 2, 3, 5, 7, 9-14.
Let's look at 2a together.

$2 + 0 = 2$

2.3 continued

Example 3: Evaluate $\lim_{h\to 0}\left(\frac{(3+h)^2-9}{h}\right)$. direct substitution does not work.

$\frac{\cancel{9}+6h+h^2-\cancel{9}}{h} = \frac{\cancel{h}(6+h)}{\cancel{h}} = 6+0 = 6$

Example 4: Find $\lim_{t\to 0}\left(\frac{\sqrt{t^2+9}-3}{t^2}\right)$. direct substitution does not work.

try conjugate

$\lim_{t\to 0} \frac{\sqrt{t^2+9}-3}{t^2}\left(\frac{\sqrt{t^2+9}+3}{\sqrt{t^2+9}+3}\right) = \frac{t^2+9-9}{t^2(\sqrt{t^2+9}+3)} = \frac{t^2}{t^2(\sqrt{t^2+9}+3)}$

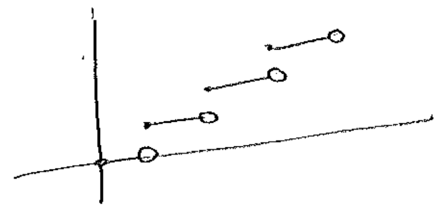
$\boxed{= \frac{1}{\sqrt{t^2+9}+3}} \Rightarrow \frac{1}{\sqrt{9}+3} = \boxed{\frac{1}{6}}$

Theorem 1:
$\lim_{x\to a} f(x)$ if and only if $\lim_{x\to a^-} f(x) = L = \lim_{x\to a^+} f(x)$.

Greatest Integer Function: is a step function that is defined by $[[x]]$ = the largest integer that is less than or equal to x. (Think of this as rounding down.)

What is $[[4]]$? $[[4.8]]$? $\left[\left[-\frac{1}{2}\right]\right]$?

4 4 -1



Theorem 2: If $f(x) \le g(x)$ when x approaches a, then $\lim_{x\to a} f(x) \le \lim_{x\to a} g(x)$.

Theorem 3: (Also called the squeeze theorem.) If $f(x) \le g(x) \le h(x)$ when x is near a (possibly not at a) and

$$\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$$

then

$$\lim_{x\to a} g(x) = L.$$

Example 5: If $2x \le g(x) \le x^4 - x^2 + 2$ for all x, evaluate $\lim_{x\to 1} g(x)$.

$\lim_{x\to 1} 2x \le \lim_{x\to 1} g(x) \le \lim_{x\to 1} x^4 - x^2 + 2$

$2 \le \lim_{x\to 1} g(x) \le \underbrace{1-1+2}_{2}$

$\therefore$ by squeeze thm $\boxed{\lim_{x\to 1} g(x) = 2}$

Homework: page 111 (4, 15-20, 22, 33, 37, 39)