Name Answer Key
Period ______ Date 2012/2013

Limit practice for Calculus 2.1-2.5

Evaluate each limit, if it exists.

1. $\lim_{x\to -3}\frac{x^2-x-12}{x+3}$

$\lim_{x\to -3}\frac{(x+3)(x-4)}{(x+3)}$

$\lim_{x\to -3} x-4 = -7$

2. $\lim_{t\to 1}\frac{t^3-t}{t^2-1}$

$\lim_{t\to 1}\frac{t(t^2-1)}{t^2-1}$

$\lim_{t\to 1} t = 1$

3. $\lim_{x\to -2}\frac{x+2}{x^2-x-6}$

$\lim_{x\to -2}\frac{x+2}{(x+2)(x-3)}$

$\lim_{x\to -2}\frac{1}{x-3} = \boxed{\frac{-1}{5}}$

4. $\lim_{t\to 9}\frac{9-t}{3-\sqrt{t}}$

$\lim_{t\to 9}\frac{(3+\sqrt{t})(3-\sqrt{t})}{3-\sqrt{t}}$

$\lim_{t\to 9} 3+\sqrt{t} = 6$

5. $\lim_{x\to 9}\frac{x^2-81}{\sqrt{x}-3}$

$\lim_{x\to 9}\frac{(x-9)(x+9)}{\sqrt{x}-3}$

$\lim_{x\to 9}\frac{(\sqrt{x}-3)(\sqrt{x}+3)(x+9)}{(\sqrt{x}-3)}$

$\lim_{x\to 9}(\sqrt{x}+3)(x+9)$

$= (3+3)(9+9)$

$6(18) = \boxed{108}$

6. $\lim_{x\to 2}\frac{x-\sqrt{3x-2}}{x^2-4}$ $\left(\frac{x+\sqrt{3x-2}}{x+\sqrt{3x-2}}\right)$

$\lim_{x\to 2}\frac{x^2-3x+2}{(x-2)(x+2)(x+\sqrt{3x-2})}$

$\lim_{x\to 2}\frac{\cancel{(x-2)}(x-1)}{\cancel{(x-2)}(x+2)(x+\sqrt{3x-2})}$

$\lim_{x\to 2}\frac{x-1}{(x+2)(x+\sqrt{3x-2})}$

$= \frac{1}{(4)(2+2)} = \boxed{\frac{1}{16}}$

Use the intermediate value theorem to show that there is a root of the given equation in the given interval for 7 and 8.

7. $x^3 - 3x + 1 = 0$, $(0, 1)$ poly continuous

$f(0) = 1$

$f(1) = -1$ $\therefore$ by IVT $\exists$ x such that $f(x) = 0$.

8. $x^3 + 2x = x^2 + 1$, $(0, 1)$ poly continuous

$x^3 - x^2 + 2x - 1 = 0$

$f(0) = -1$ $\therefore$ by IVT $\exists$ x such that $f(x) = 0$.

$f(1) = 1$

Find the limit:

9. $\lim\limits_{r\to\infty} \dfrac{r^4 - r^2 + 1}{r^5 + r^3 - r}$

$$\lim_{r\to\infty} \frac{\frac{r^4}{r^5} - \frac{r^2}{r^5} + \frac{1}{r^5}}{\frac{r^5}{r^5} + \frac{r^3}{r^5} - \frac{r}{r^5}}$$

$$= \frac{0 - 0 + 0}{1 + 0 - 0} = \frac{0}{1} = \boxed{0}$$

10. $\lim\limits_{x\to\infty} \dfrac{\sqrt{1 + 4x^2}}{4 + x}$

$$\lim_{x\to\infty} \frac{\sqrt{1/x^2 + 4x^2/x^2}}{\frac{4}{x} + \frac{x}{x}}$$

$$= \frac{\sqrt{0 + 4}}{0 + 1} = \frac{2}{1} = \boxed{2}$$

11. $\lim\limits_{x\to\infty} \dfrac{1 - \sqrt{x}}{1 + \sqrt{x}}$

$$\lim_{x\to\infty} \frac{\frac{1}{\sqrt{x}} - \frac{\sqrt{x}}{\sqrt{x}}}{\frac{1}{\sqrt{x}} + \frac{\sqrt{x}}{\sqrt{x}}}$$

$$= \frac{0 - 1}{0 + 1} = \boxed{-1}$$

need a fraction

12. $\lim\limits_{r\to\infty}\left(\sqrt{x^2 + 3x + 1} - x\right)\left(\dfrac{\sqrt{x^2+3x+1} + x}{\sqrt{x^2+3x+1} + x}\right)$

$$= \lim_{r\to\infty} \frac{x^2 + 3x + 1 - x^2}{\sqrt{x^2+3x+1} + x}$$

$$= \lim_{r\to\infty} \frac{3x + 1}{\sqrt{x^2+3x+1} + x}$$

$$= \lim_{r\to\infty} \frac{\frac{3x}{x} + \frac{1}{x}}{\sqrt{\frac{x^2}{x^2} + \frac{3x}{x^2} + \frac{1}{x^2}} + \frac{x}{x}}$$

$$= \frac{3 + 0}{\sqrt{1 + 0 + 0} + 1} = \boxed{\frac{3}{2}}$$

13. The point $P(3,1)$ lies on the curve $y=\sqrt{x-2}$. Let Q be the point $(x, \sqrt{x-2})$.

a. What is the slope of the secant line PQ (correct to six decimal places) for the following values of x?

(i)(3.1, 1.048809) (ii)(3.01, 1.004988) (iii)(3.001, 1.0004999)

.488088 .498756 .499875

(iv)(2.9, .9486833) (v)(2.99, .99498744) (vi)(2.999, .9994999)

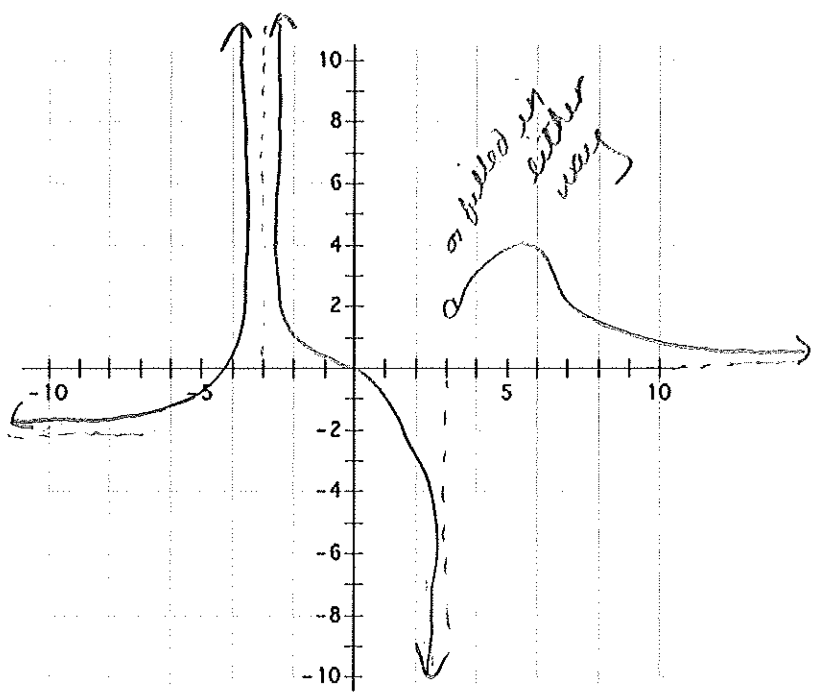
.513167 .50125629 .500125

b. Using part a, estimate the slope of the tangent line to $y=\sqrt{x-2}$ at $x = 3$. Explain your reasoning.

$m \approx 1/2$ from both the right and left side of P the slope appears to be approaching 1/2.

14. Given the following information about limits, sketch a graph which could be the graph of $y=f(x)$. Label all horizontal and vertical asymptotes.

$\lim_{x \to -\infty} f(x) = -2$, $\lim_{x \to \infty} f(x) = 0$, $\lim_{x \to -3} f(x) = \infty$, $\lim_{x \to 3^-} f(x) = -\infty$,
$\lim_{x \to 3^+} f(x) = 2$

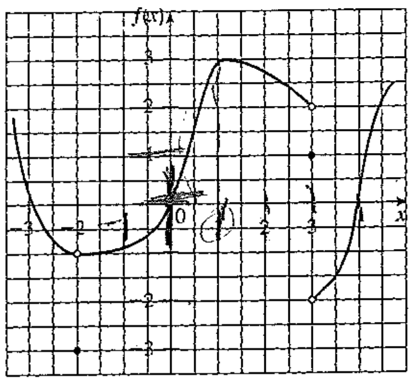


1. For the function f whose graph is given, state the value of the given quantity, if it exists.

(a) $\lim_{x \to 1} f(x)$ 3 (b) $\lim_{x \to 3^-} f(x)$ 2 (c) $\lim_{x \to 3^+} f(x)$ -2

(d) $\lim_{x \to 3} f(x)$ NE (e) $f(3)$ 1 (f) $\lim_{x \to -2^-} f(x)$ -1

(g) $\lim_{x \to -2^+} f(x)$ -1 (h) $\lim_{x \to -2} f(x)$ -1 (i) $f(-2)$ -3



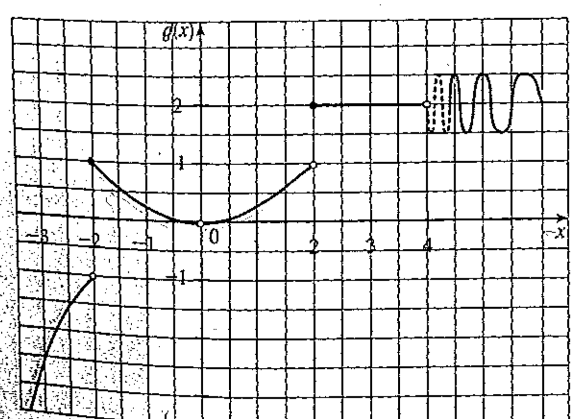
2. For the function g whose graph is given, state the value of the given quantity, if it exists.

(a) $\lim_{x \to -2^-} g(x)$ -1 (b) $\lim_{x \to -2^+} g(x)$ 1 (c) $\lim_{x \to -2} g(x)$ NE

(d) $g(-2)$ 1 (e) $\lim_{x \to 2^-} g(x)$ 1 (f) $\lim_{x \to 2^+} g(x)$ 2

(g) $\lim_{x \to 2} g(x)$ NE (h) $g(2)$ 2 (i) $\lim_{x \to 4^+} g(x)$ 2

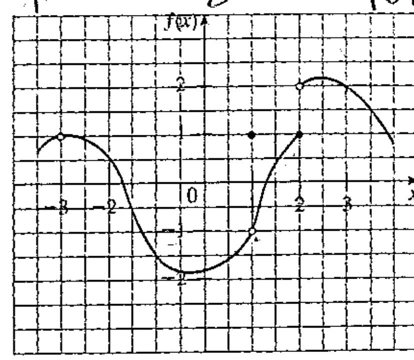
(j) $\lim_{x \to 4^-} g(x)$ 2 (k) $g(0)$ NE (l) $\lim_{x \to 0} g(x)$ 0



3. State the value of the limit, if it exists, from the given graph.

(a) $\lim_{x \to -3} f(x)$ 2 (b) $\lim_{x \to 1} f(x)$ -1 (c) $\lim_{x \to -3} f(x)$ 1

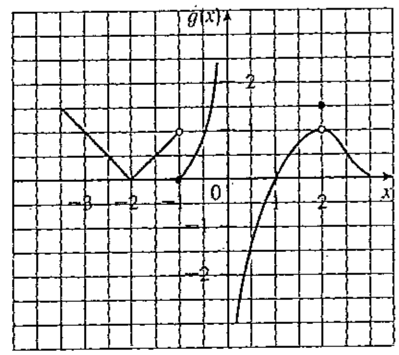
(d) $\lim_{x \to 2^-} f(x)$ 1 (e) $\lim_{x \to 2^+} f(x)$ 2 (f) $\lim_{x \to 2} f(x)$ NE



4. State the value of the limit, if it exists, from the given graph.

(a) $\lim_{x \to 1} g(x)$ 0 (b) $\lim_{x \to 0} g(x)$ NE (c) $\lim_{x \to 2} g(x)$ 1

(d) $\lim_{x \to -2} g(x)$ 0 (e) $\lim_{x \to -1^-} g(x)$ 1 (f) $\lim_{x \to -1} g(x)$ NE



5. For the function f whose graph is shown, state the following.

(a) $\lim_{x \to 3} f(x)$ ∞ (b) $\lim_{x \to 7} f(x)$ ∞ (c) $\lim_{x \to -4} f(x)$ $-\infty$

(d) $\lim_{x \to -9^-} f(x)$ ∞ (e) $\lim_{x \to -9^+} f(x)$ $-\infty$

(f) The equations of the vertical asymptotes $x=-9$ $x=-4$ $x=3$ $x=7$