Section 1.6 Inverse Functions

Hr (t)	Miles traveled $M = f(t)$
1	70
2	140
3	210
4	280

Miles traveled M	Time $t = f^{-1}(M)$
70	1
140	2
210	3
280	4

M is a one to one. In order for a function to have an inverse, it must be one to one.

One to one: A function *f* is called a one to one if it never takes on the same value twice.

Time in minutes	# of ss in classroom
1	2
2	8
3	8
4	15

#of ss in classroom	Time in minutes
2	1
8	2
8	3
15	4

This function is not a one to one, therefore, it does not have an inverse function. Why is the table on the left not a function?

Horizontal line test: A function is one-to-one if and only if no horizontal line intersects its graph more than once.

Example 1: Determine if $f(x)=x^2$ and $g(x)=x^3$ are one-to-one.

x^2 not 1-1

x^3 1-1

Inverse Function: If f is a one-to-one function with domain A and range B then its inverse function has domain B and range A and is defined by $f^{-1}(y)=x \Leftrightarrow f(x)=y$ for any y in B.

Look at the first two tables, $f(1) = 70$ so $f^{-1}(70)=1$.

Example 2: If $f(1) = 5$ and $f(3)=7$, find $f^{-1}(7)$ and $f^{-1}(5)$.

$f^{-1}(7)=3$ $f^{-1}(5)=1$

Since we think of x as the independent variable we usually reverse x and y in the inverse equation.

Cancellation Equations:

$f^{-1}(f(x)) = x$ for every x in A.

$f(f^{-1}(x)) = x$ for every x in B.

Finding the inverse function of a one-to-one:

Step 1: Write $y = f(x)$.

Step 2: Solve this equation for x in terms of y.

Step 3: To express f^{-1} as a function of x, interchange x and y. The resulting equation is $y = f^{-1}(x)$.

Example 3: Find the inverse function of $f(x) = x^3 + 2$.

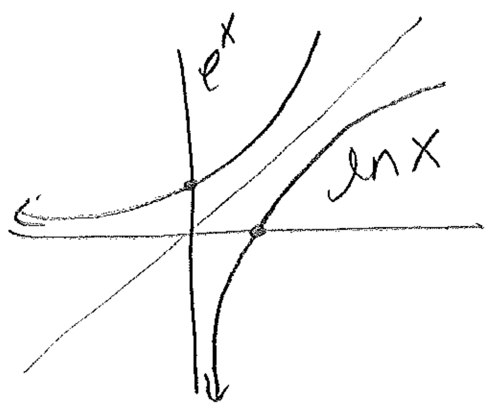
$y = x^3 + 2$

$y - 2 = x^3$

$x = \sqrt[3]{y-2}$

to express as a function of x interchange x + y so $f^{-1}(x) = \sqrt[3]{x-2}$

The graph of a functions inverse is obtained by reflecting the graph of f about the line $y = x$.



Logarithmic functions with base *a*: The inverse of $f(x) = a^x$.

$\log_a x = y \Leftrightarrow a^y = x$.

Cancellation Equations:

$\log_a(a^x) = x$ for every x in the Reals

$a^{\log_a x} = x$ for every x greater than 0

Laws of Logarithms:

1. $\log_a(xy) = \log_a x + \log_a y$
2. $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
3. $\log_a(x^r) = r\log_a x$ where r is any real number

Example 4: Use the laws of logarithms to evaluate $\log_2 96 - \log_2 6$.

$\log_2 \frac{96}{6} = \log_2 = \log_2 16 = y \quad 2^y = 2^4$

$\boxed{y = 4}$

Natural Logarithms: This is the logarithm with base *e*.

$\ln x = y \Leftrightarrow e^y = x$

Cancellation Equations:

$\ln(e^x) = x$ for all x in the Reals.

$e^{\ln x} = x$ for all x greater than 0

Example 5: Find x if $\ln x = 5$.

$e^{\ln x} = e^5$

$\boxed{x = e^5}$

Cancellation Equations:

$\ln(e^x) = x$ for all x in the Reals.

$e^{\ln x} = x$ for all x greater than 0

Example 5: Find x if $\ln x = 5$.

Example 6: Solve the equation $e^{5-3x} = 10$.

$\ln e^{5-3x} = \ln 10$

$5 - 3x = \ln 10$

$-3x = \ln 10 - 5 \Rightarrow x = -\frac{1}{3}(\ln 10 - 5)$

Example 7: Express $\ln a + ½ \ln b$ as a single logarithm.

$\ln(ab^{1/2}) = \ln(a\sqrt{b})$

Change in base formula: For any positive number a (a does not equal 1), we have

$\log_a x = \frac{\ln x}{\ln a}$

Log uses base 10

ln uses base e

$\log_2 10 = \frac{\ln 10}{\ln 2}$

Homework day 1: page 72 #3-19, 21-30.

day 2: page 72-73 #31-54, 57.