p(8)
9/19

1.5 Notes Exponential Functions

Exponential Function: has the general form of $f(x) = a^x$ where a is a positive constant.

Special cases: What happens if x = 0?

$f(x) = a^0 \quad = f(x) = 1$

What happens if x = -n?

$f(x) = a^{-n} = \frac{1}{a^n}$

$x^{3/4}$

What happens if is a rational number (x = p/q)?

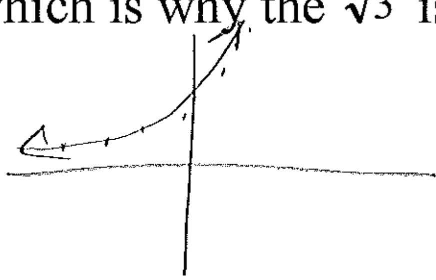
$x^{1/2}$

$f(x) = a^{p/q} = \sqrt[q]{a^p}$

What if x is irrational? Say $\sqrt{3}$ or π ?

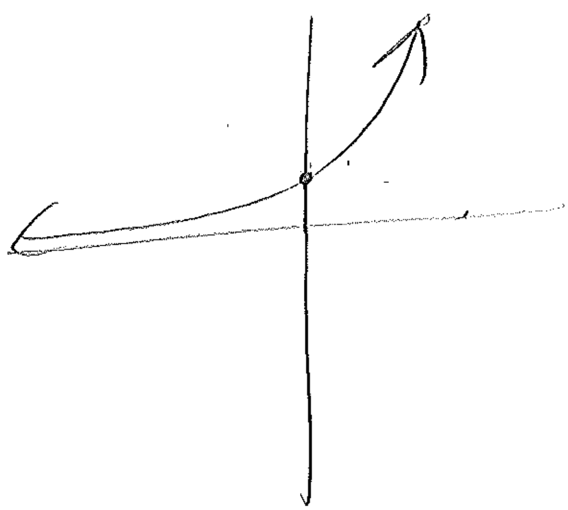
$f(x) = a^{\sqrt{3}}$

In theory, there is an infinite number of Reals between every pair of rational numbers.
The $\sqrt{3}$ is one of the Reals between 1.73 and 1.74 and between 1.732 and 1.733. This can keep going which is why the $\sqrt{3}$ is an irrational number.



Using the equation $y = 2^x$ and only rational numbers would not give a function because between every pair of rational numbers are irrational numbers so the graph would have gaps. We must then include irrational numbers in the domain to obtain the function of $y = 2^x$.

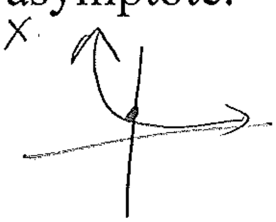
Basic graph of $y = 2^x$ is



Take a look at $y = a^x$. Consider the following situations:

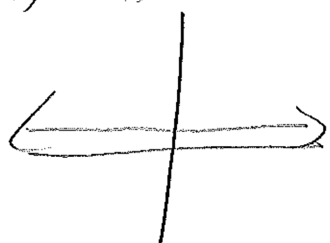
The graph of $y = a^x$ can take on three basic looks that depend on the value of a.
If a is between 0 and 1, the graph is decreasing with x = 0 being a horizontal asymptote.

$f(x) = \frac{1}{2}^{x} = 2^{-x}$

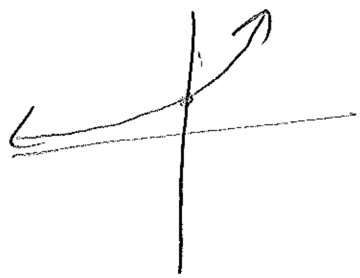


If a is equal to 1, the graph is the linear graph of y = 1.

$f(x) = 1^x = a^0 = 1$



If a is greater than 1, the graph is increasing with x = 0 being a horizontal asymptote.



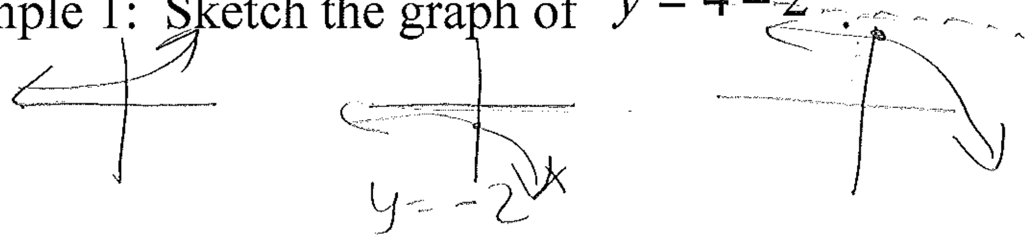
The graph of $y = a^x$ can take on three basic looks that depend on the value of a.

Laws of exponents: List them.

$x^a x^b = x^{a+b}$ $\quad (x^a)^b = x^{ab}$

$\frac{x^a}{x^b} = x^{a-b}$ $\quad (xy)^a = x^a y^a$

Example 1: Sketch the graph of $y = 4 - 2^x$.



Example 2: On your graphing calculator, graph the exponential function $f(x) = 2^x$ and the power function $g(x) = x^2$. Note the change in each especially focusing on the first quadrant. Try different window settings to get a broader picture.

Example 3: A bacterial culture starts with 500 bacteria and doubles in size every half hour.

a) How many bacteria are there after 3 hours?

$500 \; 2^6 = 32{,}000$

b) How many bacteria are there after t hours?

$500(2)^{2t} = f(t)$

c) How many bacteria are there after 40 minutes?

$\frac{2}{3}$

$f(2/3) = 500(2)^{2(2/3)} = 500(2)^{4/3}$

≈ 1260 bac

d) Graph the population function and estimate the time for the population to reach 100,000.

$f(t) = 500(2)^{2t} = y_1$

$f(t) = 100{,}000 = y_2$

$t \approx 3.82$ hrs

Half-life: means given an initial amount of any substance, how long it will take for that substance amount to deteriorate to half of its initial value.

Example 4: The *half-life* of strontium-90, ^{90}SR, is 25 years.

a) If a sample of ^{90}SR has a mass of 24 mg, find an expression for the mass *m(t)* that remains after *t* years. $m(t) = 24\left(\frac{1}{2}\right)^{t/25} = 24(2)^{-t/25}$

b) Find the mass remaining after 40 years, correct to the nearest milligram. $\approx 7.9\text{mg}$

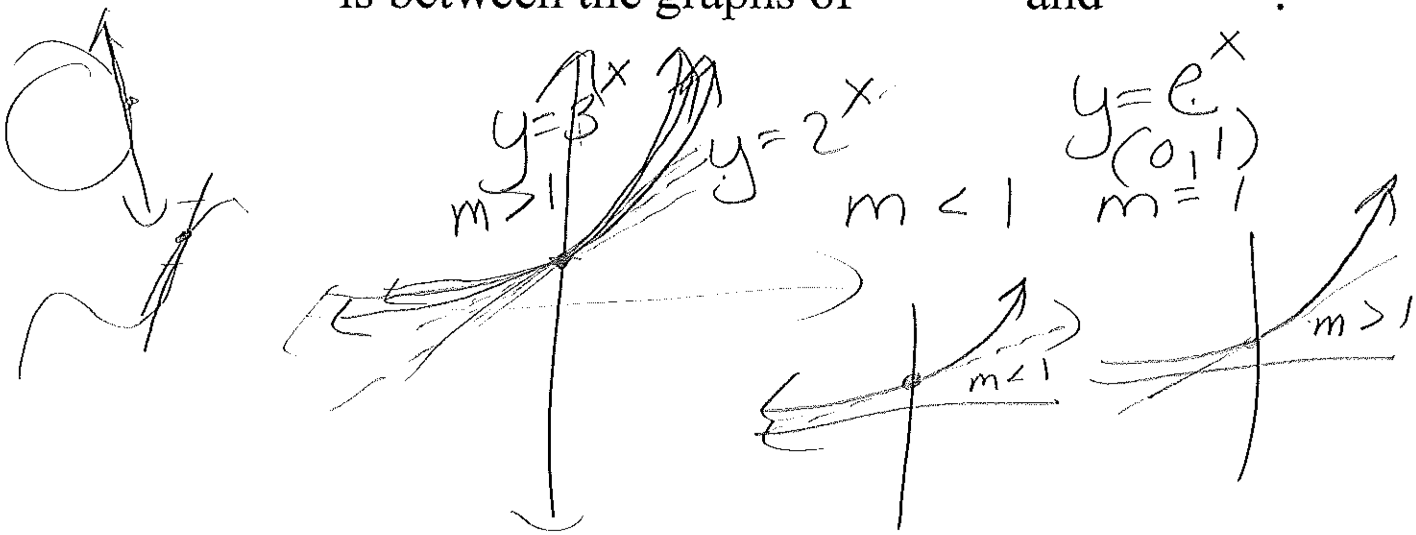
c) Use a graphing calculator to graph *m(t)* and use it to estimate the time required for the mass to be reduced to 5 mg.

$y_1 = 24(2)^{-t/25}$ [0, 100]

$y_2 = 5$ [0, 24]

$t = 56.6$ yrs

The number e is a very common base in calculus for the exponential function $y = a^x$. The graph of $y = e^x$ is between the graphs of $y = 2^x$ and $y = 3^x$.



All three of which pass through the point (0,1). It is at this point that the tangent line to each of the graphs has a slope close to 1. The graph of $y = e^x$ has a tangent line at the point (0,1) that has a slope of exactly 1 which is what make it so special.

Example 5: Graph the function $y = \frac{1}{2}e^{-x} - 1$