1.3 Notes Transformations

Translations: The key to remember with translations is that the graph is just moved up, down, left, or right. Take the graph of $f(x) = x^2$ and graph it on your calculator. Then graph each of the other graphs below noting what happens to the graph.

$f(x) = x^2 + 2$ $\quad$ $f(x) = x^2 - 2$ $\quad$ $f(x) = (x-2)^2$

$f(x) = (x+2)^2$

Suppose c is greater than 0, then

$y = f(x) + c$, shift c units up

$y = f(x) - c$, shift c units down

$y = f(x - c)$, shift c units right

$y = f(x + c)$, shift c units left.

Stretching and Reflecting: Take the graph of y = cos x and apply these transformations using 2 as the value of c (must be greater than 1):

$y = cf(x)$, stretching graph vertically by a power of C.

$y = (1/c)f(x)$, compressing graph vertically by a power of C.

$y = f(cx)$, compussing horizontally by a factor of c

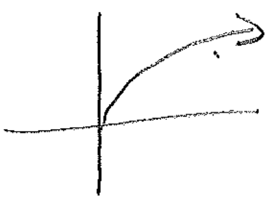
$y = f(x/c)$, stretching horizontally by a factor of c

$y = -f(x)$, refects @ the x-axis

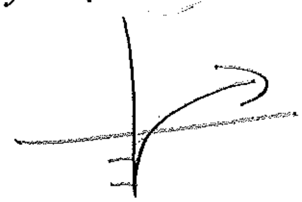
$y = f(-x)$ refects @ the y-axis

Example 1: Without using a graphing device, sketch each of these graphs:

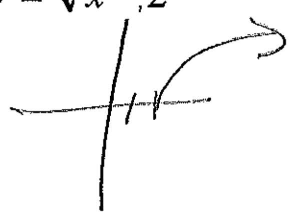
$y = \sqrt{x}$



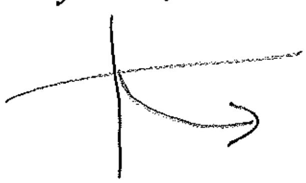
$y = \sqrt{x} - 2$



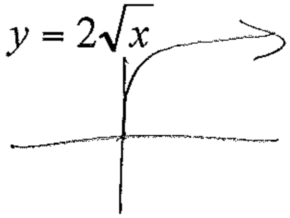
$y = \sqrt{x - 2}$



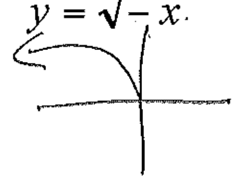
$y = -\sqrt{x}$



$y = 2\sqrt{x}$



$y = \sqrt{-x}$

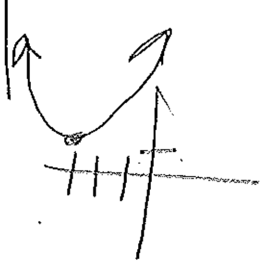


Example 2: Sketch the graph of the function $f(x) = x^2 + 6x + 10$.

$f(x) = (x^2 + 6x + 9) + 10 - 9$

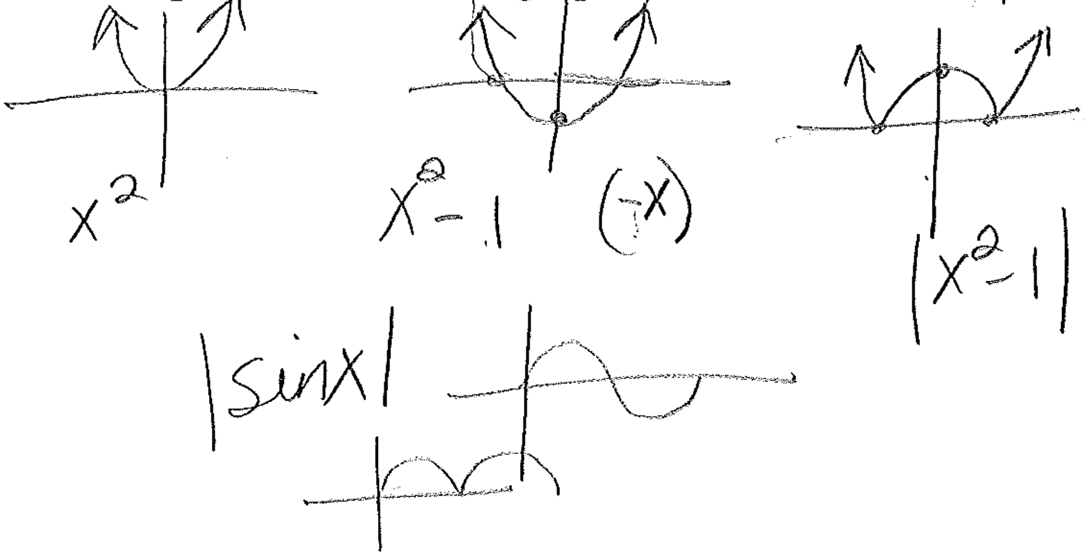
$f(x) = (x+3)^2 + 1$

x^2



$(x+3)^2$

Example 3: Sketch the graph of the function $y = |x^2 - 1|$.



Combinations of Functions: Let *f* and *g* be functions with domains A and B. Then the functions $f + g$, $f - g$, fg, and f/g are defined by:

$(f + g)(x) = f(x) + g(x)$ domain = $A \cap B$

$(f - g)(x) = f(x) - g(x)$ domain = $A \cap B$

$(fg)(x) = f(x)g(x)$ domain = $A \cap B$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ domain = $\{x \in A \cap B \mid g(x) \neq 0\}$

Example 4: If $f(x)=\sqrt{x}$ and $g(x)=\sqrt{4-x^2}$, find the four functions listed above.

$f+g=\sqrt{x}+\sqrt{4-x^2}$

$f-g=\sqrt{x}-\sqrt{4-x^2}$

$fg=\sqrt{x}\cdot\sqrt{4-x^2}=\sqrt{4x-x^3}$ $[0,2]$

$\frac{f}{g}=\frac{\sqrt{x}}{\sqrt{4-x^2}}\left(\frac{\sqrt{4-x^2}}{\sqrt{4-x^2}}\right)=\frac{\sqrt{4x-x^3}}{4-x^2}$ $[0,2)$

D: $\sqrt{x}$

$[0,\infty)$

D: $\sqrt{4-x^2}$

$4-x^2\geq 0$

$4\geq x^2$ $[-2,2]$

$2\geq |x|$

Composition of Functions: Given two functions *f* and *g*, the composite function $f\circ g$ (read f circle g) is defined by $(f\circ g)(x)=f(g(x))$.

Example 5: If $f(x)=\sqrt{x}$ and $g(x)=\sqrt{2-x}$, find each function and its domain.

a) $f\circ g$ b) $g\circ f$ c) $f\circ f$ d) $g\circ g$

a)

$f(g(x))$

$=f(\sqrt{2-x})$

$=\sqrt{\sqrt{2-x}}$

$=\sqrt[4]{2-x}$

$2-x\geq 0$

$x\leq 2$

$(-\infty,2]$

b)

$g(f(x))$

$g(\sqrt{x})$

$=\sqrt{2-\sqrt{x}}$

$2-\sqrt{x}\geq 0$

$2\geq\sqrt{x}$

$4\geq x$

$(-\infty,4]$

$\boxed{[0,4]}$

Example 4: If $f(x)=\sqrt{x}$ and $g(x)=\sqrt{4-x^2}$, find the four functions listed above.

① $f(x)+g(x) = \sqrt{x}+\sqrt{4-x^2}$ D: $[0,2]$

② $f(x)-g(x) = \sqrt{x}-\sqrt{4-x^2}$ D: $[0,2]$

③ $f(x)g(x) = \sqrt{x}\,(\sqrt{4-x^2})$ D: $[0,2]$

$= \sqrt{4x-x^3}$

④ $\dfrac{f(x)}{g(x)} = \dfrac{\sqrt{x}}{\sqrt{4-x^2}}\left(\dfrac{\sqrt{4-x^2}}{\sqrt{4-x^2}}\right) = \dfrac{\sqrt{4x-x^3}}{4-x^2}$ D: $[0,2)$

Domains $f(x)=\sqrt{x}$ $g(x)=\sqrt{4-x^2}$

$x \geq 0$ $4-x^2 \geq 0$

$[0,\infty)$ $x^2 \leq 4$ $[-2,2]$

$|x| \leq 2$

Composition of Functions: Given two functions f and g, the composite function $f \circ g$ (read f circle g) is defined by $(f \circ g)(x) = f(g(x))$.

Example 5: If $f(x)=\sqrt{x}$ and $g(x)=\sqrt{2-x}$, find each function and its domain.

a) $f \circ g$ b) $g \circ f$ c) $f \circ f$ d) $g \circ g$

a) $f(g(x))$

$f(g(x)) = \sqrt[4]{\sqrt{2-x}}$

$= \sqrt[4]{2-x}$

$2-x \geq 0$

$x \leq 2$

$(-\infty, 2]$

b) $g(f(x)) = \sqrt{2-\sqrt{x}}$

$2-\sqrt{x} \geq 0$

$2 \geq \sqrt{x}$

$4 \geq x$

$(-\infty, 4]$

$\sqrt{x}$

$[0,\infty)$

D: $[0,4]$

Homework: p 45 # 1(a,c,e,g), 2(a,b,d,e), 3, 4(a,b), 5, 7, 9-13, 15, 16, 18, 21, 23, 32, 38, 51

Homework: p 45 # 1(a,c,e,g), 2(a,b,d,e), 3, 4(a,b), 5, 7, 9-13, 15, 16, 18, 21, 23, 32, 38, 51