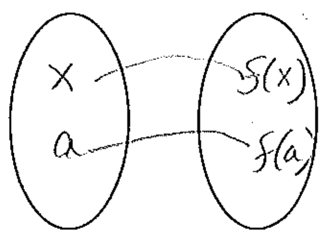
Notes Master

Notes for Section 1.1 Day 1

Functions:

- A **function** f is a rule that assigns to each element x in a set A exactly one element, called $f(x)$.
- **Domain:** the set A (independent variable) x-axis
- **Range:** the set of all $f(x)$ as x varies throughout the domain (dependent variable) y-axis



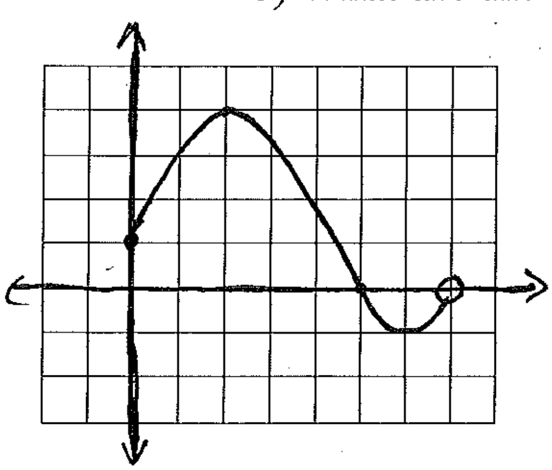
Domain Range

- 4 ways to represent a function: Verbally, numerically, visually, algebraically

Example 1: The graph of a function f is shown below.

a) Find $f(1)$ and $f(5)$.

b) What are the domain and range of f?



a) $f(1) = 3$

$f(5) = 0$

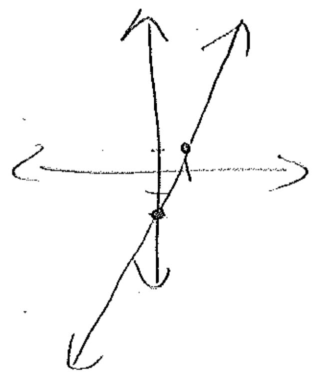
b) Domain $[0, 7)$

Range $[-1, 4]$

- **Vertical Line Test:** A curve in the xy-plane is the graph of a function of x if and only if no vertical line intersects the curve more than once.

Example 2: Sketch the graph and find the domain and range of each function.

a) $f(x) = 3x - 2$

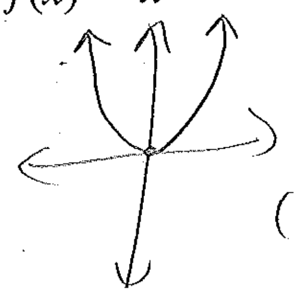


$D = \mathbb{R}$

or

$(-\infty, \infty)$

b) $f(x) = x^2$



$D = \mathbb{R}$

or

$(-\infty, \infty)$

Example 3: Draw a graph based on my description of drawing bath water.

① Turn on faucet 1 min

② Put in plug 3 min

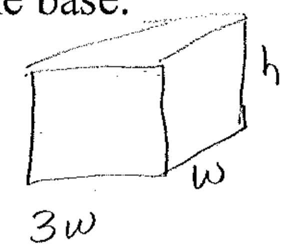
③ Get in 5 min

④ pull plug 1 min

⑤ get out

Example 4: A rectangular storage container with an open top has a volume of 21 cubic meters. The length of its base is three times the width. Material for the base costs \$9 per square meter; material for the sides is \$6 per square meter. Express the cost of materials as a function of the width of the base.

$h = \frac{7}{w^2}$



$V = 3w^2h = 21\ m^3$

$C = 9(3w^2) + 6(2wh) + 6(6wh)$

$C = 27w^2 + 48wh$

$C(w) = 27w^2 + 48w\left(\frac{7}{w^2}\right)$

$$\boxed{C(w) = 27w^2 + \frac{336}{w}}$$

Homework for day 1: page 21 (1, 2, 4-8, 10, 19, 21, 23)

Notes for Section 1.1 Day 2

Example 1: Find the domain of each function.

a) $f(x) = \sqrt{x-1}$

$x - 1 \geq 0$

$x \geq 1$

$D: [1, \infty)$

b) $f(x) = \dfrac{3}{x^2 - x}$

$x^2 - x \neq 0$

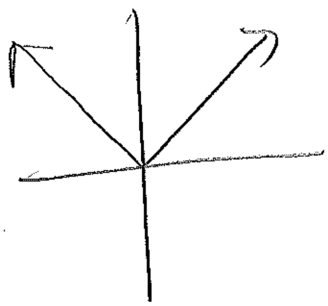
$x(x-1) \neq 0$

$x \neq 0$ or $x \neq 1$

$D: (-\infty, 0) \cup (0, 1) \cup (1, \infty)$

- **A piecewise function:** is defined by different formulas in different parts of their domains.

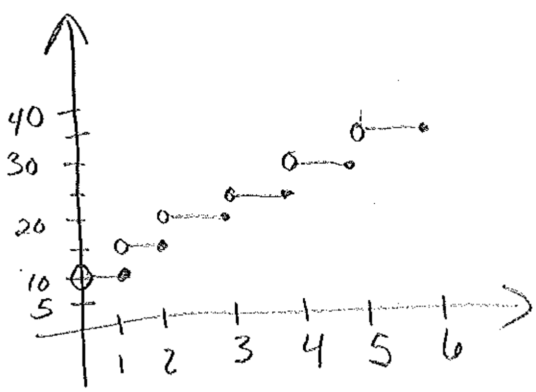
Example 2: Sketch the graph of the absolute value function $f(x) = |x|$.



$$f(x) = \begin{cases} -x & \text{if } x \leq 0 \\ x & \text{if } x > 0 \end{cases}$$

- **Step Functions:** are similar to piecewise functions however they jump from one value to another.

Example 3: When I have a baby sitter, I pay \$10 for anytime up to 1 hour and \$5 for each hour after that. Graph this data and define the function.



Homework day 2: page 23 (29, 32, 33, 35, 38, 40, 43, 45, 47, 51, 52, 55, 57)

Notes for Section 1.1 Day 3

The difference quotient: $\dfrac{f(a+h)-f(a)}{h}$

Example 1: If $f(x)=2x^2-5x+1$ and $h\neq 0$, evaluate $\dfrac{f(a+h)-f(a)}{h}$.

① $f(a+h) = 2(a+h)^2 - 5(a+h)+1 = 2a^2+4ah+2h^2-5a-5h+1$

② $f(a+h)-f(a) = \cancel{2a^2}+4ah+2h^2-\cancel{5a}-5h+\cancel{1}-(\cancel{2a^2}-\cancel{5a}+\cancel{1})$

$= 4ah+2h^2-5h$

③ $\dfrac{f(a+h)-f(a)}{h} = \dfrac{\cancel{h}(4a+2h-5)}{\cancel{h}} = \boxed{4a+2h-5}$

Another difference quotient: $\dfrac{f(x)-f(a)}{x-a}$

Example 2: If $f(x)=\dfrac{x+3}{x+1}$ find $\dfrac{f(x)-f(1)}{x-1}$.

$= \dfrac{\dfrac{x+3}{x+1} - \dfrac{1+3}{1+1}}{x-1} \Rightarrow \dfrac{\dfrac{x+3}{x+1} - 2}{x-1}$

$\Rightarrow \dfrac{1}{x-1}\left(\dfrac{x+3-2x-2}{x+1}\right) \Rightarrow \dfrac{-x+1}{(x-1)(x+1)} = \dfrac{-1(x-1)}{(x-1)(x+1)}$

$= \boxed{\dfrac{-1}{x+1}}$

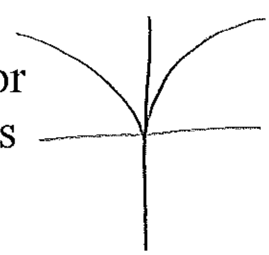
Functional notation:

Example 3: If $f(x) = 2x^2 - 4x + 1$, find f(a+1) and f(2a).

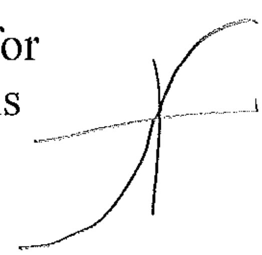
$$f(a+1) = 2(a+1)^2 - 4(a+1) + 1$$
$$= 2a^2 + \cancel{4a} + 2 - \cancel{4a} - 4 + 1$$
$$\boxed{= 2a^2 - 1}$$

$$f(2a) = 2(2a)^2 - 4(2a) + 1$$
$$\boxed{f(2a) = 8a^2 - 8a + 1}$$

Even Functions: If a function f satisfies $f(-x)=f(x)$ for every x in the domain it is called an even function. (This graph is symmetric with respect to the y-axis)



Odd Functions: If a function f satisfies $f(-x)= -f(x)$ for every x in the domain it is called an odd function. (This graph is symmetric about the origin)



Example 4: Determine whether each of the following function is even, odd, or neither.

a) $f(x) = x^5 + x$ b) $g(x) = 1 - x^4$ c) $h(x) = 2x - x^2$

$f(-x) = (-x)^5 + (-x)$

$-x^5 + -x$

$-1(x^5 + x)$

$= -f(x)$

$\therefore$ odd

$g(-x) = 1 - (-x)^4$

$= 1 - x^4$

$= g(x)$

$\therefore$ even

$h(-x) = 2(-x) - (-x)^2$

$= -2x - x^2$

$= -1(2x + x^2)$

$\therefore$ neither

Homework day 3: page 23 (23-25, 27, 65-69, and difference quotient worksheet)