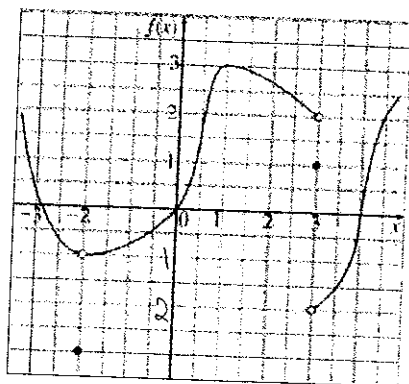


Name Answer Key P:

Use the graph below for the following four questions:



1. For the function whose graph is given above, determine  $\lim_{x \rightarrow -3^+} f(x)$ .

- (A) -3      (B) -2      (C) -1      (D) 0  
 (E) 1      (F) 2      (G) 3      (H) Does not exist

2. For the function whose graph is given above, determine  $\lim_{x \rightarrow 3} f(x)$ .

- (A) -3      (B) -2      (C) -1      (D) 0  
 (E) 1      (F) 2      (G) 3      (H) Does not exist

3. For the function whose graph is given above, determine  $\lim_{x \rightarrow -2^-} f(x)$ .

- (A) -3      (B) -2      (C) -1      (D) 0  
 (E) 1      (F) 2      (G) 3      (H) Does not exist

4. For the function whose graph is given above, determine  $\lim_{x \rightarrow 2^-} f(x)$ .

- (A) -3      (B) -2      (C) -1      (D) 0  
 (E) 1      (F) 2      (G) 3      (H) Does not exist

5. Find the value of the limit  $\lim_{x \rightarrow 1} \frac{2x^2 + x - 3}{x^2 - x}$ .

- (A) 5      (B) 4      (C) 3      (D) 2  
 (E)  $\frac{7}{2}$       (F)  $\frac{3}{2}$       (G)  $\frac{1}{2}$       (H) 0

6. Find the value of the limit  $\lim_{x \rightarrow \infty} \frac{7 + 3x}{4 - x}$ .

- (A)  $\frac{3}{4}$       (B) 3      (C) 7      (D)  $-\frac{3}{4}$   
 (E) -3      (F)  $-\frac{7}{4}$       (G) -7      (H)  $\frac{7}{4}$

7. Find the value of the limit  $\lim_{x \rightarrow \infty} \frac{4x^2 - 3x + 5}{7 + 2x - 2x^2}$ . -2. *Factor*
- (A)  $-\frac{1}{2}$                       (B) 1                      (C) 2                      (D)  $\frac{1}{2}$   
 (E) -1                      (F)  $-\frac{4}{7}$                       (G) -2                      (H)  $\frac{4}{7}$
8. Find the value of  $x$  at which the curve  $y = \frac{2x+1}{2x^2+7x+3}$  has a vertical asymptote.  $\frac{2x+1}{(2x+1)(x+3)}$
- (A) -4                      (B) -3                      (C) -2                      (D) -1  
 (E)  $-\frac{1}{2}$                       (F) 1                      (G) 2                      (H) 3
9. Find the value of  $x$  at which the curve  $y = \frac{x^2 - 16}{x^2 - 5x + 4}$  has a vertical asymptote.  $\frac{(x-4)(x+4)}{(x-4)(x-1)}$
- (A) -4                      (B) -3                      (C) -2                      (D) -1  
 (E) 0                      (F) 1                      (G) 2                      (H) 3
10. Find the derivative of  $f(t) = \sqrt{t^5} + \sqrt[3]{t^2}$ .  $t^{1/5} + t^{2/5} \Rightarrow \frac{1}{5}t^{-4/5} + \frac{2}{5}t^{-3/5}$
- (A)  $5t^{-1/2} + 2t^{-3/2}$                       (B)  $\frac{5}{2}t^{1/2} - \frac{2}{5}t^{3/5}$                       (C)  $\frac{5}{2}t^{3/2} + \frac{2}{5}t^{-3/5}$                       (D)  $\frac{2}{5}t^{3/2} + \frac{5}{2}t^{-3/5}$   
 (E)  $5\sqrt{t^4} + 2\sqrt[3]{t}$                       (F)  $\frac{5}{2}t^4 - \frac{2}{5}t$                       (G)  $\frac{3}{2}t^{5/2} - \frac{2}{3}t^{3/2}$                       (H) None of these
11. Find the derivative of  $f(x) = e^x(x^2 + 2)$ .  $e^x(x^2+2) \cdot 1 + 2xe^x = e^x(x^2+2x+2)$
- (A)  $2xe^x$                       (B)  $e^x(2x+2)$                       (C)  $2x^2e^{x-1}$                       (D)  $e^x(x^2+2x)$   
 (E)  $x^2e^{x-1}$                       (F)  $e^x(x^2+2x+2)$                       (G)  $e^x(2x+2)$                       (H) None of these
12. Find the derivative of  $f(x) = \frac{x}{x^2+3}$ .  $\frac{x^2-3}{(x^2+3)^2}$
- (A)  $\frac{1}{2x+3}$                       (B)  $\frac{-3+x^2}{(x^2+3)^2}$                       (C)  $\frac{3-x^2}{(x^2+3)^2}$                       (D)  $\frac{3-x^2}{(x^2+3)}$   
 (E)  $\frac{3}{(x^2+3)^2}$                       (F)  $\frac{1}{2x}$                       (G)  $\frac{3-x}{(x^2+3)^2}$                       (H) None of these
13. If  $f(x) = \frac{e^x}{x}$ , find  $f''(x)$ .  $\frac{xe^x - e^x}{x^2} = \frac{e^x(x-1)}{x^2}$
- (A)  $\frac{e^x(x+4)}{x^4}$                       (B)  $\frac{e^x(x^2-1)}{x^4}$                       (C)  $\frac{e^x(x^2+x)}{x^4}$                       (D)  $\frac{e^x(x^2+3)}{x^4}$                       (E)  $\frac{e^x(x-2)}{x^3}$                       (F)  $\frac{e^x(x^2+5x)}{x^3}$   
 (G)  $\frac{e^x(x^2-2x+2)}{x^3}$                       (H)  $\frac{e^x(x^3-4x^2+3)}{x^3}$
- Handwritten work for Q13:*  
 $f'' = \frac{(e^x + xe^x - e^x)x^2 - 2x(xe^x - e^x)}{x^4}$   
 $f'' = \frac{e^x(x^3 - 2x^2 + 2x)}{x^4}$   
 $f'' = \frac{e^x(x^2 - 2x + 2)}{x^3}$

14. Find the derivative of  $f(x) = x^2 \cos x$ .

$2x \cos x - x^2 \sin x$

(A)  $2x \cos x$

(B)  $2x \cos x - x^2 \sin x$

(C)  $-2x \sin x$

(D)  $2x \cos x + x^2 \sin x$

(E)  $2x \sin x$

(F)  $3x^2 \cos(x^3)$

(G)  $-x^2 \sin x$

(H) None of the above

15. Find the derivative of  $f(x) = \cos x - \sin x$ .

$-\sin x - \cos x$

(A)  $\sin x + \cos x$

(B)  $-\sin x + \cos x$

(C)  $\sin x - \cos x$

(D)  $-\sin x - \cos x$

(E)  $2 \sin x$

(F)  $2 \cos x$

(G)  $-2 \sin x$

(H)  $-2 \cos x$

16. Let  $f(x) = x \ln(x^2 - 3)$ . Find the value of  $f'(2)$ .

$\ln(x^2-3) + x \cdot \frac{2x}{x^2-3} = f'$

(A) 0

(B) 2

(C) 4

(D) 6

(E) 8

(F) 10

(G) 12

(H) 14

$f'(2) = \ln(4-3) + 2 \cdot \frac{4}{4-3} = 1 + 8 = 9$

17. Let  $f(x) = \ln(\sin^2 x + 1)$ . Find the value of  $f'(\frac{\pi}{4})$ .

(A) 0

(B)  $\frac{1}{2}$

(C)  $\frac{2}{3}$

(D) 1

(E)  $\frac{3}{2}$

(F)  $\sqrt{2}$

(G) 2

(H) 3

18. Find the absolute minimum and maximum values of the function  $f(x) = 4x^3 - 15x^2 + 12x + 7$  on the closed interval  $[0, 3]$ .

(A) 0, 3

(B) 0, 5

(C) 3, 5

(D) 3, 9.75

(E) 3, 16

(F) 5, 7

(G) 7, 16

(H) 5, 10.25

$12x^2 - 30x + 12 = 0$   
 $3x^2 - 10x + 4 = 0$   
 $3(x-2)(x-\frac{2}{3}) = 0$   
 $x = 2, \frac{2}{3}$

19. Find the minimum and maximum values of  $y = x^3 - 9x + 8$  on the interval  $[-3, 1]$ .

(A) 8,  $8 + 6\sqrt{3}$

$3x^2 - 9 = 0$

(B) 0,  $8 + 6\sqrt{3}$

(C) 0, 8

$3x^2 - 9 = 0$

(D) 8,  $8 + \sqrt{3}$

(E) 0,  $8 + \sqrt{3}$

$2x - \sqrt{3} = 0$

(F)  $8 + \sqrt{3}$ ,  $8 + 6\sqrt{3}$

(G)  $\sqrt{3}$ , 8

$- \sqrt{3}$

(H)  $\sqrt{3}$ ,  $8 + 6\sqrt{3}$

$f'(x) = 0$   
 $x^2 - 3 = 0$   
 $x = \pm \sqrt{3}$   
 $f(\sqrt{3}) = 18.4$

20. Find the most general antiderivative of the function  $f(x) = \frac{2+x^3}{x^3}$ .

- (A)  $\frac{2+3x^2}{3x^2} + C$       (B)  $\frac{2x + \frac{1}{4}x^4}{x^4} + C$       (C)  $-\frac{1}{x^2} + 1 + C$       (D)  $-\frac{1}{x^2} + C$   
 (E)  $\frac{1}{x^2} + x + C$       (F)  $\frac{1}{x^2} + 1 + C$       (G)  $-\frac{1}{x^2} + x + C$       (H)  $\frac{2}{x^2} + C$

21. Find the value of the integral  $\int_1^3 \frac{1}{x^2} dx$ .

- (A)  $\frac{2}{3}$       (B)  $\frac{1}{2}$       (C)  $-\frac{1}{2}$       (D)  $-\frac{1}{3}$   
 (E)  $-\frac{2}{3}$       (F) 1      (G)  $\frac{1}{3}$       (H) -1

22. Find the value of the integral  $\int_1^2 \frac{x^2-1}{x} dx$ .

- (A)  $\frac{1}{2}$       (B) 1      (C)  $\frac{3}{2}$       (D) 2  
 (E)  $\frac{1}{2} \ln 2$       (F)  $1 + \ln 2$       (G)  $\frac{3}{2} - \ln 2$       (H)  $2 + \ln 2$

23. Find the value of the integral  $\int_0^1 \frac{x^2}{(x^3+1)^2} dx$ .

- (A)  $\frac{3}{4}$       (B) 2      (C)  $\frac{3}{7}$       (D)  $\frac{7}{3}$   
 (E)  $\frac{1}{6}$       (F)  $\frac{3}{2}$       (G)  $\frac{2}{3}$       (H) 1

24. Find the value of  $\int_e^{e^2} \frac{(\ln x)^2}{x} dx$ .

- (A)  $\ln 2$       (B)  $\frac{1}{2} \ln 2$       (C)  $\frac{1}{2}$       (D)  $\frac{3}{2}$   
 (E) 1      (F)  $1/(\ln 2)$       (G) 0      (H)  $\frac{7}{3}$

25. Find the value of the integral  $\int_0^1 \frac{e^x}{e^x+1} dx$ .

- (A)  $e+1$       (B)  $\ln(e-1)$       (C)  $\frac{e-1}{2}$       (D)  $\ln \frac{e+1}{2}$   
 (E)  $\frac{1}{2} \ln(e-1)$       (F)  $\frac{e+1}{2}$       (G)  $\frac{1}{2} \ln(e+1)$       (H)  $e-1$

26. Evaluate the following integrals:

(a)  $\int x^3 \cos(x^4+1) dx = \frac{1}{4} \sin(x^4+1) + C$       (c)  $\int \frac{\cos(1/x)}{x^2} dx = \sin\left(\frac{1}{x}\right) + C$

(b)  $\int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx$

(d)  $\int \frac{\cos(\arctan x)}{1+x^2} dx$

$2 \sin \sqrt{x} + C$

$\sin(\arctan x) + C$

27. Evaluate the following integrals:

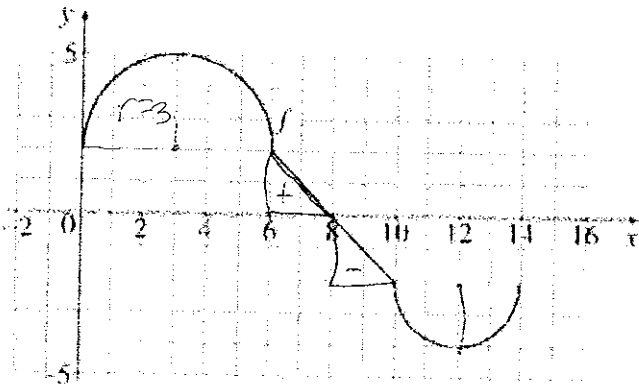
(a)  $\int x^3 e^{(x^4+1)} dx = \frac{e^{x^4+1}}{4} + C$

(b)  $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2e^{\sqrt{x}} + C$

(c)  $\int \frac{e^{(1/x)}}{x^2} dx = -e^{1/x} + C$

(d)  $\int \frac{e^{(\arctan x)}}{1+x^2} dx = e^{\arctan x} + C$

28. Find the net area of the given region from 0 to 14.



$$\frac{\pi \cdot 3^2}{2} + \frac{\pi \cdot 2^2}{2} - \frac{(9+4)\pi}{2} = \frac{13\pi}{2}$$

29. Find the derivative of each of the following functions.

a.  $f(x) = \frac{2}{3} \cos^3(3x)$

$$f(x) = \frac{2}{3} (\cos(3x))^3$$

$$f'(x) = 2 (\cos(3x))^2 \cdot 3(-\sin(3x))$$

$$f'(x) = -6 \sin(3x) \cos^2(3x)$$

b.  $f(x) = \ln(5x^2 + 4)$

$$f'(x) = \frac{10x}{5x^2+4}$$

c.  $f(x) = \ln(\sin^2 x)$

$$f'(x) = \frac{2 \sin x \cos x}{\sin^2 x}$$

$$f'(x) = \frac{2 \cos x}{\sin x} = 2 \cot x$$

d.  $f(x) = \frac{2}{3} e^{-x^5}$

$$f(x) = \frac{2}{3} (-5x^4) e^{-x^5}$$

$$f'(x) = -\frac{10}{3} x^4 e^{-x^5}$$

30. Use the first derivative test to find the intervals of increase and decrease for the

function  $f(x) = \frac{3}{2}x^4 - 5x^3 - \frac{9}{2}x^2 + 5$ .

$$f'(x) = 4\left(\frac{3}{2}x^3\right) - 15x^2 - \frac{18}{2}x \quad 6x^3 - 15x^2 - 9x$$

$$(-\infty, -\frac{1}{2}) \quad (-\frac{1}{2}, 0) \quad (0, \frac{3}{2}) \quad (\frac{3}{2}, \infty)$$

$\downarrow$     $\uparrow$     $\downarrow$     $\uparrow$

$$3x(2x^2 - 5x - 3)$$

$$3x(2x+1)(x-3)$$

$0, -\frac{1}{2}, \frac{3}{2}$

31. Find the intervals where  $f$  is concave up and concave down for

$f(x) = 2x^4 - 3x^3 + x - 10$ .

$$f' = 8x^3 - 9x^2 + 1 \quad X = 0, \frac{3}{4}$$

$$f'' = 24x^2 - 18x$$

$$f'' = 3x(8x - 6)$$

$$(-\infty, 0) \quad (0, \frac{3}{4}) \quad (\frac{3}{4}, \infty)$$

$\uparrow$     $\downarrow$     $\uparrow$

$CU$     $CD$     $CU$

32. Find each indefinite integral.

a.  $\int \frac{5x^2 - \sqrt{x}}{x} dx \quad \int 5x - x^{-1/2} dx$

$$\frac{5}{2}x^2 - 2x^{1/2} + C$$

b.  $\int 3x^2 + 4x - 21 dx$

$$\frac{3x^3}{3} + \frac{4x^2}{2} - 21x + C$$

$$x^3 + 2x^2 - 21x + C$$

c.  $\int 5 \sin \theta - 3 \cos \theta d\theta$

$$-5 \cos \theta - 3 \sin \theta + C$$

33. Solve the differential equations.

a.  $\frac{dZ}{dx} = 20x^4 - 3x^3, Z(0) = 3$

$$Z = \frac{20x^5}{5} - \frac{3x^4}{4} + C$$

$$Z = 4x^5 - \frac{3}{4}x^4 + C$$

b.  $\frac{dQ}{dy} = 4 \sin y + 2y, Q(0) = 7$

$$Q = -4 \cos y + y^2 + 7$$